

**Intermediate scientific report of the contract PN-III-P1-1.1-PD-2016-0331**  
regarding the implementation of the project during january - september 2019

**A. Overall summary of the project**

The research team and the roles of the members who carried out research activities within the project "*Nonlinear observers-based control structures applied to mechatronics systems*", financing contract no. 164/10.10.2018, with the deposit code PN-III-P1-1.1-PD-2016-0331, <https://szedlak-stinean.wixsite.com/nobsmeecs>, is the one nominated in the funding application: Lect. Dr. Ing. Alexandra-Iulia Szedlak-Stînean, the project leader and Prof. Dr. Ing. Radu-Emil Precup, the mentor.

The *main objectives* pursued during the contract in 2019 are set out in the project implementation plan and are grouped in the form of the following stages and activities:

**O1: Stage 1 - The development of the theoretical framework which allows the implementation of the modern control solutions** - continuation from 2018. The activities carried out to achieve this goal are the following:

*1.1. Obtaining current theoretical knowledge regarding observers.* To accomplish this activity the analysis of current theoretical research – state-of-the-art on theoretical research – and the possibilities of improving modern control solutions were considered. Details are presented in the study from section B.

*1.2. Studying the practical applications of various types of observers.* In order to accomplish this activity, the realization and classification of observers was studied with focus on mechatronic systems. Details are also presented in the study from section B.

**O2: Stage 2 - The design and implementation of the proposed modern control solutions** pursued within the project in 2019 (January - September 2019) has been completed and is grouped in the form of the following activity:

*2.1. Development of observers through simulations.* In order to carry out this activity, the analysis of theoretical research and the classification of observers specific to mechatronic systems from section B, were taken into account. It should be noted that in the published scientific papers, experimental results are included in addition to the simulation results, highlighted by the electric drive system with time-varying inputs with brushless DC motor and with the help of laboratory equipment Model 220 Industrial Plant Emulator (MIPE220). The details are presented in the techniques from section C.

**O3: Exploitation and dissemination of research results.** The activities set out for this objective given in section E refers to:

*3.1. Participation and presentation of research results at important conferences in the field.* The result was 4 scientific papers published in the volumes of conferences indexed in international databases (INSPEC, IEEE Xplore, Scopus, DBLP) and 2 scientific papers are currently being published.

*3.2. The intermediate research report of the contract PN-III-P1-1.1-PD-2016-0331* only for the period january - september 2019, due to the suspension of the project starting with 01.10.2019, the reason for the interruption being the entry into maternity leave.

For **2019**, the deliverables proposed in the initial implementation plan are: the present **intermediate research report**, **1 scientific paper** published in a specialized journal (this deliverable could not be achieved due to the suspension of the contract, but will be achieved in the last stage of the contract research) and **1 scientific paper** published in the volume of a specialized conference indexed in international databases.

The **main results** obtained in 2019 are:

- **4 scientific papers** published in the volumes of conferences indexed in international databases (INSPEC, IEEE Xplore, Scopus, DBLP) and **2 scientific papers** are currently being published.

- **1 scientific report.**

**Remarks:**

1. The bibliographic study related to sections B and C is presented in section D, and the results obtained during 2019 are presented in section E.

2. In this research report, the relationships and figures were taken from the papers prepared by the research team, which is why annotations may appear in English. Each time, however, the necessary clarifications are made to make the interpretations as clear as possible.

3. The results obtained are also mentioned on the project web page, <https://szedlak-stinean.wixsite.com/nobsmeccs>, where all the information related to the project will be included.

**B. The development of the theoretical framework which allows the implementation of the modern control solutions. Study on acquiring current theoretical knowledge regarding observers and analyzing the practical applications of various types of observers**

The research conducted during 2010-2019 concerns the analysis, synthesis and modeling of mechatronics systems based on electric drives. Modern control solutions are reviewed and developed dedicated towards controlling the speed and position of electric drives with variable parameters. The control strategies of servo-systems should provide very good dynamic performance and steady-state conditions for the developed control systems. In this sense, the main research was oriented towards the development of new control solutions dedicated to speed and position control of three mechatronics applications:

1. The electric drive system which wraps a strip with constant linear velocity on a drum - electric drive system with variable parameters.
2. The electric drive system with time-varying inputs with a brushless motor.
3. The laboratory equipment - Model 220 Industrial Plant Emulator.

Mechatronics systems are successfully used in many industrial and non-industrial applications because of their simple and robust structure. By constantly evolving, these systems exhibit increasing performance ensuring functional and applicative versatility, intelligence and flexibility. These features are also provided by the control system which has to be able to adapt to external conditions at all times and to provide the necessary informations to the hierarchical control (Isermann, 2005; Koch et al., 2006; Hehenberger et al., 2006; Pabst, 2006; Bishop, 2007; Dragos, 2011). Mechatronic applications incorporate:

- Mechanical subsystem (the controlled process);
- Electronic interface subsystems (the execution and measuring elements);
- Control subsystem.

The degree of complexity of the structure and of the control subsystem of a mechatronic application differs from one application to another and may include relatively simple and advanced control structures. The following points of view may be considered in the development of mechatronic applications (Dragos, 2011):

- accepting a simplified system representation;
- establishing the operating performance;
- control of specific measurements, including state estimation;
- generating dynamic behavior in special situations;
- development of advanced algorithms;



- detecting and diagnosing operating errors.

The design of control systems is important because of the very good performance specifications imposed in various fields. The specifications become more restrictive if several operating conditions of the processes are involved as the variable parameters (for example, in the case of speed and position control besides the variable moment of inertia situations, the variable reference input, correlated with various particular operating conditions, additional operating conditions may occur, such as oscillations that appear on the mechanical side). Furthermore, the variable parameters can appear when the controller design is carried out using simplified benchmark-type models of complicated process models. These problems can be solved in two ways: by developing advanced nonlinear models or by implementing a robust control based on state observers technique. The implementation of the second solution in servo-systems leads to high system performances. In this context, the development of advanced control solutions is a relatively difficult problem which requires a detailed preliminary study of the process, particularly the parameters variability and of its structural properties. As a continuation of previous research, in this project (PD 164/10.10.2018) the approach oriented on the analysis, synthesis, modeling and development of modern control solutions, such as nonlinear observers dedicated to certain mechatronics applications is proposed. Two types of observers can be used: linear and nonlinear. From the category of **linear observers**, the most common are **full-order observers** known as the Luenberger type observers and **reduced-order observers**. Among the nonlinear ones we mention: **Kalman-extended filters**, **sliding mode observers**, **fuzzy** and **neural network observers** (Luenberger, 1966; Ackermann, 1972; Friedland, 1996; Gajic și Lelic, 1996; Stefani et al., 2002; Preitl et al., 2009).

Luenberger (Luenberger, 1966) was the first to introduce and solve the problem of designing observers for linear control systems. Thau (Thau, 1973) was first to propose one of the central problems in control systems literature, the problem of designing observers for nonlinear control systems. Many methods of process control use the state reaction principle. In most cases, the only measurements available to the system are the input and output measurements, which is why it is necessary from this information to build the state model chosen to develop the command. A state constructor (Dorf și Bishop, 2005; Preitl și Precup, 2008; Preitl et al., 2009) is a system having as input the inputs and outputs of the real process, and as output an estimate of the state of this process. In the hypothesis of linearity of the process model, the basic structure of the estimator is always the same, but its realization will depend on the chosen context: continuous or discrete, deterministic or stochastic. If the model is deterministic, the state constructor will be called an observer (or estimator). If the model is a stochastic type, then the constructor will be called a filter. The estimation problem requires the construction, for a deterministic model of the process, of a system defined by its state equation, for which the output provides an estimate of the real state of the process. This estimate implies an error that must tend to zero; when this property is satisfied, the estimator or observer is called asymptotic (Dorf and Bishop, 2005; Preitl and Precup, 2008; Preitl et al., 2009). An observer is very useful for implementing feedback stabilization or feedback regulation of nonlinear control systems due to the fact that it is essentially an estimator for the state of the system. The states estimation problem is of important strategic significance for the process of controlling and monitoring of many technological processes. In order to control and optimize a process, knowing some physical state variables provides useful information. This is the case of many widely diffused process control strategies. Therefore by including an appropriate state estimator, the difficulty caused by the presence of unknown states can be solved. For this reason, the development of suitable algorithms to perform the estimation has been the focus of many researchers attention and for this purpose, in order to estimate state variables from the available measurements, several techniques have been developed and introduced. Over the past three decades there has been significant research done on the nonlinear observer design problem. Depending on the mathematical structure of the process model, there is a wide variety of possible estimators that

can be used (Soroush, 1997; Mouyon, 1997; Garcia et al., 2000; Aghannan și Rouchon, 2003; Kinsey and Whitcomb, 2007; Ghanes et al., 2013; Magnis and Petit, 2016).

The filtering process aims to determine the estimation of system variables when the environment in which the process takes place presents random perturbations. Two points of view can be used to address this issue: the first one is Wiener's who uses the frequency description and the second one is Kalman's using the temporal description. In both cases, an optimal system (filter) is determined in order to minimize the error variation between the real variable and its estimation (Wiener, 1949; Boulfelfel et al., 1994; Brown and Hwang, 1996). The probabilistic estimation method that is the most well-known and commonly used for linear systems is the **Kalman filter (KF)** and for the nonlinear systems its extension, **the Extended Kalman Filter (EKF)**. Although the KF in the case of highly nonlinear processes becomes unstable and has severe limitations, when employed for linear processes provides an efficient method for estimating the states and thus it also minimizes the mean of the squared error. Even if a precise model of the considered system is unknown the filter supports the estimation of past, present and future states (Biagiola and Solsona, 2006; Lendek et al., 2008; Beyhan et al., 2013; Habibullah and Lu, 2015). Regarding the nonlinear estimation techniques that have been developed until now, the EKF among other nonlinear ones based on linearization techniques is one of the most widely diffused observers. For the EKF approach, in order to obtain the estimator gain a Riccati equation must be solved. Even though the EKF could be a good choice when the assumptions are satisfied, in order to have a good trade-off between the measurement noise and the input uncertainty, it can still fail in many cases. Because of the fact that the noise model is often unknown and it can only be assumed, wrong noise assumptions can lead to biased or even divergent estimates. The purpose of the observer is to process these measurements and any available informations regarding the initial state of the system and to offer an estimate of the current state of the system. The obtained estimation should improve with additional measurements and, ideally, in the absence of noise, it should converge to the true state value. The advantage of using EKF is the lack of stationary error at low speeds. The disadvantage of the method arises from the complexity and large volume of computing, for the implementation the use of processors with high computing power being necessary. Another drawback is the need to fully know the noise sources in the system in order to determine the covariance matrices. For most applications the EKF is still the approach that is more practical and offers the most robustness (Brown and Hwang, 1996; Simon, 2006; Biagiola and Solsona, 2006; Lendek et al., 2008; Beyhan et al., 2013; Habibullah and Lu, 2015).

The estimation of states and of the possible uncertain parameters, based on a dynamic system model and a sequence of noisy measurements, is required by a wide range of problems in decision making, control, and monitoring. The design of an observer for a nonlinear system requires the description of the system in a state-space form. There is no generic method yet developed for the design of an observer valid for all nonlinear system types. From this point of view *fuzzy control* is more pragmatic due to the specific decision-making mode in the command development. To this end, a rather qualitative linguistic characterization of the situation in which the process evolves and of the intervention decision is developed; this decision is then transposed into an adequate quantitative intervention (Preitl și Precup, 2008; Preitl et al., 2009). Any nonlinear system can be approximated on a compact set with an arbitrary accuracy by the generic nonlinear state-space model that is provided by Takagi-Sugeno (TS) fuzzy systems (Takagi and Sugeno, 1985). The interest of using the approach based on the representation of nonlinear systems through TS models (Takagi and Sugeno, 1985; Taniguchi et al., 2001; Tanaka and Wang, 2001) is build on the fact that once the TS fuzzy models are obtained, some analysis and design tools developed in the theory of linear systems (Huang, 2005; Lee et al., 2013; Li et al., 2014) which facilitates the observation and/or synthesis of the controller for complex nonlinear systems can be used. However, many papers have been done in order to investigate the problem of nonlinear observers synthesis and its application to the dynamic systems described by fuzzy TS systems. In particular, in (Tanaka et al.,



1998, Tanaka and Wang, 2001, Bergsten et al., 2002, Ichalal et al., 2007, Lendek et al., 2010), the problem of the TS fuzzy observer for explicit dynamic models in continuous-time and in discrete-time has been addressed. In implicit cases, there are more continuous-time papers (Taniguchi et al., 2000, Marx et al., 2007, Ilhem et al., 2012; Essabre et al., 2014; Soulami et al., 2015) than for discrete-time (Wang et al., 2012; Estrada-Manzo et al., 2014). Moreover, many other works regarding the explicit design of the observer, called the *Proportional Integral Observer*, have also been proposed for the TS implicit models. These results are based on the singular value decomposition approach and on a generalized inverse matrix and take into account the output matrix without nonlinear terms (Marx et al., 2007; Hamdi et al., 2013). The analysis and design for a general nonlinear system represented by a fuzzy model becomes much easier due to the fact that there are well-established methods and algorithms that can be used to design **fuzzy observers**. For TS fuzzy systems there have been several types of observers developed, such as Thau-Luenberger observers (Beyhan et al., 2013; Tanaka et al., 1998), reduced order observers (Beyhan et al., 2013; Bergsten et al., 2002) and sliding mode observers (Beyhan et al., 2013; Palm și Bergsten, 2000). The observers design methods generally lead to a linear matrix inequalities (LMIs) feasibility problem. The stability analysis for a large number of rules eventually becomes unsolvable due to the fact that the system complexity grows exponentially with the number of antecedents (Lam et al., 2013). By employing observers, the non-measurable states can be estimated, analyzed and used for the control of nonlinear systems.

Variable Structure Systems (VSS) are one of the most promising techniques for controlling electrical drives, due to good robustness and performance in case of parametric variations, of perturbations and load variations, respectively due to the compensation of the nonlinearities of the controlled process and their simple implementation. The primary feature that distinguishes the sliding mode systems as an independent class of automatic control systems is the fact that during transient processes the system structure changes. In VSS theory, the most attention is paid to systems with sliding modes. They are based on a specific type of control law that causes a *sliding mode control* or *sliding mode regime* in the system. **Sliding mode observers** use the remarkable VSS properties and are successfully integrated into servo-systems. These observers use the equivalent command to estimate the equivalent disturbance. In general, their structures are closely related to the mathematical models used in the controlled process (Spurgeon, 2008; Mercorelli, 2015; Apaza-Perez et al., 2016). These have the ability to minimize the error between the measured process output and the output of the observer that ensures that the observer produces a set of state estimates that are precisely proportional with the actual output of the process. The sliding mode observer design method consists in determining a switching gain. One restriction is the fact that in order to achieve the estimation, the outputs have to lie on specified sliding surface. In addition, performance is rarely guaranteed, particularly when the outputs are corrupted with noise. Similarly to a Kalman filter, the sliding mode observers have attractive noise resilience properties. In comparison to linear observers, the principle advantage that the sliding-mode observers have is that they are insensitive to the unknown inputs while in sliding, and additionally they can be utilized in order to reconstruct unknown inputs which could be a combination of system disturbances, faults or nonlinearities (Spurgeon, 2008; Mercorelli, 2015; Apaza-Perez et al., 2016).

Building upon the previously discussed state-of-the-art, the **main objective of this proposal is to develop the necessary tools, modern control solutions and theoretical framework for later multi-purpose applications related to the observer-based control of mechatronics systems**. The control systems should benefit from the advantages of the continued analytical development and implementation of dynamic model-based nonlinear observers that promise to improve the performance of different types of mechatronics systems.

The following **difficulties have been identified that may affect the proposed approach**:

(a) the stability analysis for some of the proposed approaches may be difficult to perform no matter what techniques are used (e.g. error linearization, frequency domain techniques, Lyapunov techniques).

(b) environments where several constraints appear (e.g. constraints imposed to the control signal and/or control signal rate, constraints imposed to the controlled output, etc).

(c) the extension of the approaches is expected to work for smooth nonlinear processes that can be well approximated with linear systems in the vicinity of nominal operating points.

The construction of nonlinear observers still provides an open research field because advances in this area often face many obstacles, such as, for example, the very restrictive conditions that have to be satisfied, uncertainty in the performance and robustness and/or poor estimation results. Depending on what type of observer is being used there are other limitations, for example, in case of Luenberger-based observers, design is always based on the perfect knowledge of the system parameters; in case of finite-dimensional system observers, the convergence factor depends strongly on the operating conditions and for artificial intelligence-based observers it may be difficult and time consuming for online implementation and also, the artificial intelligence elements must first be adapted to the system.

Modern control approaches as the **EKF**, **fuzzy observers** and **sliding mode observers** are utilized to estimate states and unknown variables in a nonlinear process in the eventual presence of disturbances or noise. Prior to developing the observer's equation, most researchers developed observers based on the mathematical model of the systems and used the first principles model leading to the fact that model-based approaches are employed by most observer designs. Although the observer gain and its estimation error dynamics are also significant, the evaluation of an observer designs success is based on its ability to estimate the difficult-to-measure states with acceptable convergence rates and with approximately zero estimation errors.

Due to the fact that observers can only be designed for observable systems, the first important consideration before designing the observer is to verify the observability condition of the system, which in turn has influence upon the observer's feasibility conditions. In reconstructing unmeasurable state variables the concept of observability is central, thus explaining the need of observers to estimate unknown states prior to developing control laws and the fact that not all states are available directly through online measurements. Observers that can estimate the state variables can be designed once the observability conditions are fulfilled by the system dynamics. Therefore it is highly important to choose an appropriate observer for each different mechatronic system. Before that, the states that have to be estimated and the initial conditions have to be clearly defined. The stability of all control systems and stability conditions for the model-based observers will be analyzed using suitable techniques. Thereafter in order to determine the performance of the proposed observers, tests are run that compare the actual values with the estimates. The validation of the control approaches will be done first by numerical simulation scenarios and, after that, experimentally on the various mechatronics applications available in the laboratories. The further development and improvement of the control approaches can be pursued independently. The proposed nonlinear observers will be designed to deal with nonlinearities and also with constraints on different control variables.

**The potential impact to the scientific field** may be significant because through new concepts and employed approaches, a new way for the use of highly advanced control designs in mechatronics applications is open, which is very useful for implementing feedback stabilization or feedback regulation of nonlinear control systems. It is clear that nonlinear observer design is still an open area for research, efforts being made to broaden and adapt the proposed techniques in order to widen the classes of nonlinear systems to which they may apply.

**The potential impact of the project in the scientific, social, economic or cultural environment** is straightforward since the investigated topics can lead to automated tools for controller design and tuning. Although there is a wide range of possibilities for creating new themes



for state-of-the-art research, noteworthy is also the impact in the socio-economic environment with directly applicative directions. In the project all mechatronics applications tackled are interdisciplinary and multidisciplinary themselves, with special focus on those applicable cost-effective training systems in the fields of robotics, automation and process control.

**C. Design and implementation of modern control solutions. Control structures based on linear / nonlinear observers intended for mechatronic applications.**

### C1. Estimation of acceleration and speed related to time-varying electric drive systems with brushless DC motors (BLDC) using different types of observers

The electric drive systems with Brushless Direct Current (BLDC) motors (BLDC drives) have experienced a fast and complex multidisciplinary development as a result of advances in various fields such as (Yedamale, 2003; Hansson, 2004; Baldursson, 2005; Nasar and Boldea, 2005): power electronics, modern automated control techniques, powering electric vehicles (e.g., cars, electric bicycles, electric scooters), numerical computing systems based on microelectronics with a high degree of integration, industrial automation, medical equipments and robotic instrumentation (e.g., improving sleep for apnea suffers, providing mobile breathing assistance, preventing blood clots in hospitalized patients), etc. In control applications which involve electric drives, it is often necessary to estimate derivatives of measurable states, for example, estimating the speed ( $\omega$ ) and acceleration ( $\varepsilon$ ) from position information ( $\theta_m$ ), which is why in this paper five design approaches are proposed and developed: (1) an observer of  $m^{th}$  order derivatives with the filtration technique (OFT), (2) an extended Luenberger state observer with  $\theta_m$  and  $M_e$  as inputs (ELSO+Me), (3) an extended Luenberger state observer having only the  $\theta_m$  as input (ELSO-Me), (4) a sliding mode observer (SMO), and (5) a Kalman filter observer (KFO).

The controlled process is represented by a BLDC drive with current control loop. For the BLDC drive, the switching process occurs at discrete moments that can be detected with Hall sensors (exemplified in Fig. C1.1). For a three-phase BLDC motor (star connection), the current always passes through two phases, which is convenient because the unpowered phase can be monitored to determine the effect of the electromotive voltage at the zero crossing point (Măgureanu și Vasile, 1990; Hanselman, 2006; Dragu, 2009; Marshall și Stutz, 2012). The control system based on the rotor position can provide the correct switching sequence. The Matlab/Simulink block diagram of the BLDC drive (Stinean, 2014) is illustrated in Fig. C1.1.

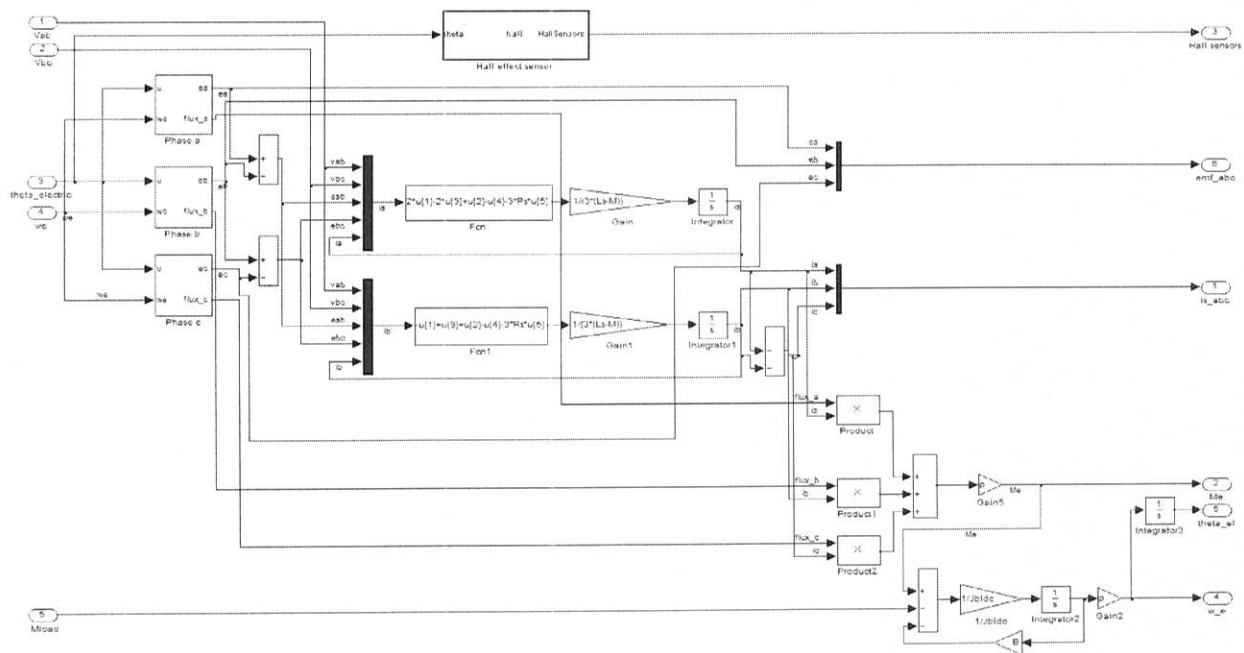


Fig. C1.1. Matlab/Simulink block diagram of BLDC drive.

The mathematical model (MM) of the three-phase BLDC motor in matrix form is (Hanselman, 2006; Marshall și Stutz, 2012; Stinean, 2014):

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} R_a & 0 & 0 \\ 0 & R_b & 0 \\ 0 & 0 & R_c \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} L_a & M_{ab} & M_{ac} \\ M_{ba} & L_b & M_{bc} \\ M_{ca} & M_{cb} & L_c \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix}. \quad (C1.1)$$

where:  $L_a, L_b, L_c$  and  $R_a, R_b, R_c$  – the phase inductance and resistance,  $M_{ab}, M_{ac}, M_{ba}, M_{bc}, M_{ca}, M_{cb}$  – the mutual inductance,  $e_a, e_b, e_c$  – the electromotive voltage,  $i_a, i_b, i_c$  and  $v_a, v_b, v_c$  – the phase currents and voltages. Considering that  $L_a = L_b = L_c = L_s = L$ ,  $R_a = R_b = R_c = R_s = R$ ,  $M_{ab} = M_{bc} = M_{ca} = M_{ba} = M_{cb} = M_{ac} = M$  and knowing that  $i_a + i_b + i_c = 0$ , then the new form MM is:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} R & 0 & 0 \\ 0 & R & 0 \\ 0 & 0 & R \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} L-M & 0 & 0 \\ 0 & L-M & 0 \\ 0 & 0 & L-M \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} + \begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix}. \quad (C1.2)$$

Eq. (C1.2) leads to:

$$\begin{aligned} v_{ab} &= R(i_a - i_b) + (L - M) \frac{d}{dt}(i_a - i_b) + e_{ab}, \\ v_{bc} &= R(i_b - i_c) + (L - M) \frac{d}{dt}(i_b - i_c) + e_{bc}. \end{aligned} \quad (C1.3)$$

The value of the electromotive voltage is dependent on the rotor position and speed. The equation of the electromotive voltage in matrix form is expressed in:

$$\begin{bmatrix} e_a \\ e_b \\ e_c \end{bmatrix} = \omega_m K_e \begin{bmatrix} F(\theta_e) \\ F(\theta_e - 2\pi/3) \\ F(\theta_e + 2\pi/3) \end{bmatrix} \quad (C1.4)$$

and the electromagnetic torque  $M_e$  of the BLDC motor is given in (Hanselman, 2006; Marshall și Stutz, 2012; Stinean, 2014):

$$M_e = (e_a i_a + e_b i_b + e_c i_c) / \omega_m, \quad (C1.5)$$

where:  $\omega_m$  – rotor mechanical speed,  $\theta_e$  – rotor electric position,  $\theta_m$  – (rotor) mechanical position,  $K_e$  – constant of electromotive voltage. The expression of the electromagnetic torque used in the movement equation is:

$$M_e = J_{BLDC} \frac{d}{dt} \omega_m + B \omega_m + M_{Load} \quad (C1.6)$$

where  $M_{Load}$  – load torque. Using the information presented above, the MM of the three-phase BLDC motor is of the fourth order, having as states variables  $i_a, i_b, \theta_m$  and  $\omega_m$ . In this context, the process dynamics can be divided into two subsystems: *the electromagnetic subsystem (ES)* and *the mechanical subsystem (MS)*. The dynamic equations associated with the two subsystems in state-space form are:

$$\begin{aligned} ES: \begin{bmatrix} \dot{i}_a \\ \dot{i}_b \end{bmatrix} &= \begin{bmatrix} \frac{-R}{L-M} & 0 \\ 0 & \frac{-R}{L-M} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix} + \begin{bmatrix} \frac{2}{3(L-M)} & \frac{1}{3(L-M)} \\ \frac{-1}{3(L-M)} & \frac{1}{3(L-M)} \end{bmatrix} \begin{bmatrix} v_{ab} - e_{ab} \\ v_{bc} - e_{bc} \end{bmatrix} \\ Me &= \frac{e_a i_a + e_b i_b + e_c i_c}{\omega_m}, \end{aligned} \quad (C1.7)$$

$$MS: \begin{bmatrix} \dot{\theta}_m \\ \dot{\omega}_m \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & \frac{-B}{J_{BLDC}} \end{bmatrix} \begin{bmatrix} \theta_m \\ \omega_m \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \frac{1}{J_{BLDC}} M_e + \begin{bmatrix} 0 \\ \frac{-1}{J_{BLDC}} \end{bmatrix} M_{Load},$$

where  $x_{ES} = [i_a \ i_b]^T$  (here,  $T$  represents matrix transposition) are the states of the ES and  $x_{MS} = [\theta_m \ \omega_m]^T$  are the states of the MS. The five design approaches presented below will use only the corresponding dynamic MS model.



### C1.1. Observer of $m^{th}$ order derivatives with filtration technique (OFT)

The ideal  $m^{th}$  order derivative is characterized by a pure anticipatory dynamic, so it is not practically achievable. A possible solution is to use an  $n^{th}$  order low-pass filter for the measured  $\theta_m$ , connected in series with a pure derivation operator, respecting the constraint  $m \leq n$ . The values  $m=2$  and  $n=3$  are set due to the fact that  $d^2\theta_m/dt^2 = d\omega_m/dt = \varepsilon$ , where  $\varepsilon$  represents the acceleration. Using the substitution  $x = [x_1 \ x_2 \ x_3]^T = [\theta_m \ \omega_m \ \varepsilon]^T$  the observable canonical form used in the observer structure in Fig. C1.1.1 is:

$$\underbrace{\begin{bmatrix} \dot{\hat{\theta}}_m \\ \dot{\hat{\omega}}_m \\ \dot{\hat{\varepsilon}} \end{bmatrix}}_{\dot{\hat{x}}} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{bmatrix}}_A \underbrace{\begin{bmatrix} \hat{\theta}_m \\ \hat{\omega}_m \\ \hat{\varepsilon} \end{bmatrix}}_{\hat{x}} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ a_0 \end{bmatrix}}_B \theta_m \quad \text{or} \quad \hat{\varepsilon} = \frac{a_0 s^2}{s^3 + a_2 s^2 + a_1 s + a_0} \theta_m \quad (C1.1.1)$$

Naming this structure an observer is justified due to the fact that Fig. C1.1.1 shows a correction based on the output estimation error, although for the estimation using the filtering technique this idea was not the starting point. It is expected that the dynamic response of the estimate is slower because through the coefficient  $a_0$  there is only one direct correction term based on the position error. The terms  $a_1$  and  $a_2$  appear as state feedbacks and not the estimation error. The observer design uses the pole placement method. For the imposed poles (real negative ones)  $P_\sigma = \{p_1, p_2, p_3\} = \{-120, -230, -310\}$  the observer coefficients are:

$$a_2 = -(p_1 + p_2 + p_3), a_1 = p_1 p_2 + p_1 p_3 + p_2 p_3, a_0 = -p_1 p_2 p_3. \quad (C1.1.2)$$

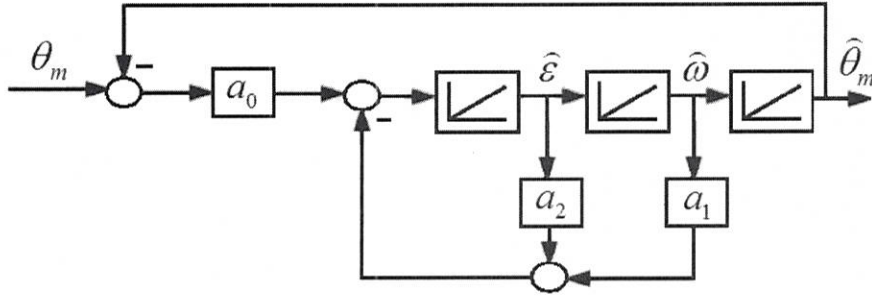


Fig. C1.1.1. OTF block diagram.

### C1.2. Extended Luenberger state observer with $\theta_m$ and $M_e$ as inputs (ELSO+Me)

A solution to compute the state estimates  $\hat{\omega}$ ,  $\hat{\varepsilon}$  and  $\hat{M}_{Load}$  is to use an extended Luenberger state observer (ELSO) having as predictive correction term the mechanical position error,  $\Delta\theta = \theta_m - \hat{\theta}_m$  and as direct input the electromagnetic torque,  $M_e$ . For  $M_{Load}$  an exogenous model of step signal class is used. In this particular case of ELSO, the state-space equations are (Luenberger, 1966; Andreescu, 1999):

$$\underbrace{\begin{bmatrix} \dot{\hat{\theta}}_m \\ \dot{\hat{\omega}}_m \\ \dot{\hat{M}}_{Load} \end{bmatrix}}_{\dot{\hat{x}}} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & \frac{-B}{J_{BLDC}} & \frac{-1}{J_{BLDC}} \\ 0 & 0 & 0 \end{bmatrix}}_A \underbrace{\begin{bmatrix} \hat{\theta}_m \\ \hat{\omega}_m \\ \hat{M}_{Load} \end{bmatrix}}_{\hat{x}} + \underbrace{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}_B \tilde{M}_e + \underbrace{\begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix}}_L (\theta_m - \hat{\theta}_m), \quad (C1.2.1)$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_C \underbrace{\begin{bmatrix} \hat{\theta}_m \\ \hat{\omega}_m \\ \hat{M}_{Load} \end{bmatrix}}_{\hat{x}}$$

where  $x = [\theta_m \ \omega_m \ M_{Load}]^T$  and the parameters  $l_1$ ,  $l_2$  and  $l_3$  of the observer gain are calculated using the pole placement method, knowing that the pair (A,C) is observable. The ELSO+Me structure is presented in Fig. C1.2.1 and the observer parameters are:

$$\begin{aligned} l_1 &= -p_1 + p_2 + p_3 - B/J_{BLDC}, \\ l_2 &= p_1 p_2 + p_1 p_3 + p_2 p_3 - l_1 (B/J_{BLDC}), \quad l_3 = J_{BLDC} p_1 p_2 p_3. \end{aligned} \quad (C1.2.2)$$

The observer coefficients depend as expected on the MS parameters (especially on  $J_{BLDC}$ ). These coefficients determine the dynamics of the observer convergence speed. The idea of using all physical states as observer inputs leads to an observer with improved performance with minimal dynamic delays.

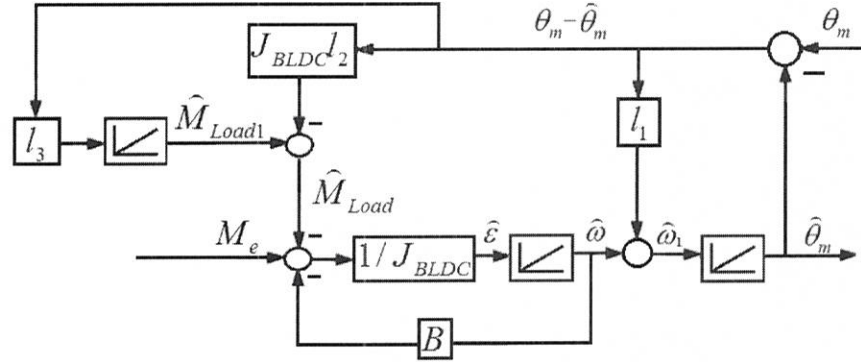


Fig. C1.2.1. ELSO+Me block diagram.

### C1.3. Extended Luenberger state observer with $\theta_m$ as input (ELSO-Me)

Another solution to compute the estimates  $\hat{\omega}$  and  $\hat{\varepsilon}$  is to use an ELSO considering only the mechanical position error term, without any other input. The motion model is chosen with  $\varepsilon = const$ , a case commonly encountered in constant acceleration with constant electromagnetic torque and in constant steady-state motion regimes. For this case of ELSO, (here,  $x = [\theta_m \ \omega_m \ \varepsilon]^T$ ) the state-space equations are (Luenberger, 1966; Andreescu, 1999):

$$\begin{aligned} \begin{bmatrix} \dot{\hat{\theta}}_m \\ \dot{\hat{\omega}}_m \\ \dot{\hat{\varepsilon}} \end{bmatrix} &= \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}}_A \begin{bmatrix} \hat{\theta}_m \\ \hat{\omega}_m \\ \hat{\varepsilon} \end{bmatrix} + \underbrace{\begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix}}_L (\theta_m - \hat{\theta}_m), \\ y &= \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_C \begin{bmatrix} \hat{\theta}_m \\ \hat{\omega}_m \\ \hat{\varepsilon} \end{bmatrix}. \end{aligned} \quad (C1.3.1)$$

The ELSO-Me block diagram is illustrated in Fig. C1.3.1. The observer compensator design  $L = [l_1 \ l_2 \ l_3]^T$  is also done with the pole placement method and the observer parameters are:

$$l_1 = -p_1 + p_2 + p_3, \quad l_2 = p_1 p_2 + p_1 p_3 + p_2 p_3, \quad l_3 = -p_1 p_2 p_3. \quad (C1.3.2)$$

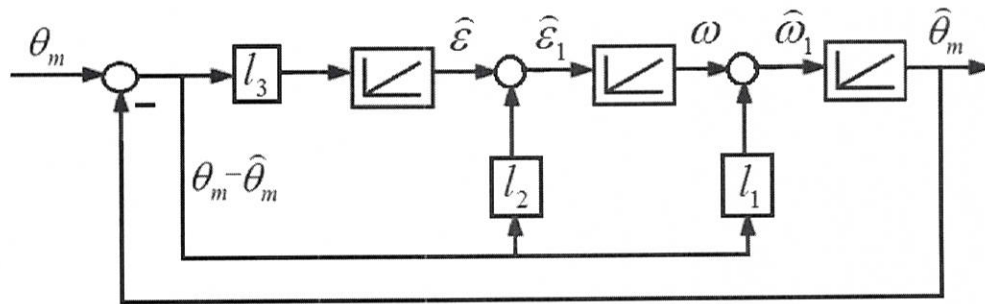


Fig. C1.3.1. ELSO-Me block diagram.



#### C1.4. Sliding mode observer (SMO)

This solution for computing  $\hat{\omega}$  and  $\hat{\varepsilon}$  involves the use of an SMO. As the name of this observer suggests, the SMO design approach consists in determining a switching gain (Spurgeon, 2008; Shtessel et al., 2014). One constraint is the fact that in order to achieve the estimation, the outputs have to lie on a specified sliding surface. For the electric drive system with BLDC motor, the nominal linear system is described in (C1.7) (the MM related to MS, here,  $C=[1 \ 0]$  and  $D=0$ ) where the matrix dimensions are  $A \in R^{n \times n}$ ,  $B \in R^{n \times m}$ ,  $C \in R^{p \times n}$ ,  $p \geq m$ , with the matrices B and C of full rank, and the pair (A, C) is observable. In order for the output to appear as component of a new state vector, it is convenient to introduce a coordinate transformation, so, taking this into account we consider the change of coordinates  $x \rightarrow T_c x$ ,  $T_c = [N_c^T \ C]^T$ , where the submatrix  $N_c \in R^{n \times (n-p)}$  spans the null space of C and  $\det(T_c) \neq 0$ . By applying the change of coordinates  $x \rightarrow T_c x$ , the matrices of the similar system are:

$$T_c A T_c^{-1} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad T_c B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \quad C T_c^{-1} = \begin{bmatrix} 0 & I_p \end{bmatrix} \quad (C1.4.1)$$

The canonical form for the nominal system can be rewritten as:

$$\begin{aligned} \dot{x}_1 &= A_{11}x_1 + A_{12}y + B_1u, \\ y &= A_{21}x_1 + A_{22}y + B_2u, \end{aligned} \quad (C1.4.2)$$

where  $T_c x = [x_1 \ y]^T$ . In this context, the SMO proposed by Utkin is (Spurgeon, 2008; Shtessel et al., 2014):

$$\begin{aligned} \dot{\hat{x}}_1 &= A_{11}\hat{x}_1 + A_{12}\hat{y} + B_1u + L v_i, \\ \dot{\hat{y}} &= A_{21}\hat{x}_1 + A_{22}\hat{y} + B_2u - v_i, \end{aligned} \quad (C1.4.3)$$

where  $(\hat{x}_1, \hat{y})$  are the state and output estimates,  $L \in R^{(n-p) \times p}$  is a gain matrix and  $v_i = M \operatorname{sgn}(\hat{y}_i - y_i)$  is a discontinuous injection term with  $M \in R_+$ . The error dynamical system is:

$$\begin{aligned} \dot{e}_1 &= A_{11}e_1 + A_{12}e_y + L v_i, \quad e_1 = \hat{x}_1 - x_1, \\ \dot{e}_y &= A_{21}e_1 + A_{22}e_y - v_i, \quad e_y = \hat{y} - y. \end{aligned} \quad (C1.4.4)$$

An ideal sliding motion will take place on the surface  $S = \{(e_1, e_y) : e_y = 0\}$  and after some finite time for all subsequent time  $\dot{e}_y = e_y = 0$ . The error system defined by (C1.4.4) gives the following relation for the reduced order sliding motion:

$$\dot{e}_1 = (A_{11} + LA_{21})e_1, \quad (C1.4.5)$$

which depending on  $L$ , represents a stable system, so  $e_1 \rightarrow 0$  and consequently  $\hat{x}_1 \rightarrow x_1$ . The SMO block diagram is given in Fig. C1.4.1.

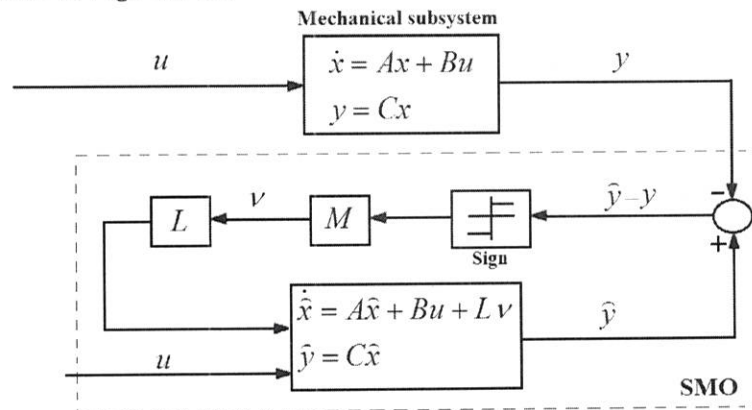


Fig. C1.4.1. SMO block diagram.

### C1.5. Kalman filter observer (KFO)

KFO refers to a probabilistic estimation approach, and is the most well-known and commonly used observer for linear systems. Although the KFO in the case of highly nonlinear processes becomes unstable and has severe limitations, when employed for linear processes provides an efficient approach for estimating the states and thus it also minimizes the mean square error (MSE). Even if an accurate model of the considered system is unknown the filter supports the estimation of past, present and future states (Brown și Hwang, 1996; Lendek et al., 2008; Habibullah și Lu, 2015). As well as the other observers discussed above, KFO will also use the dynamic model corresponding to the MS (here, with  $C=[1 \ 0]$  and  $D=0$ ). The dynamical state-space model can be written as:

$$\begin{aligned}\dot{x} &= A x + B u + G w, \\ y &= C x + D u + H w + v,\end{aligned}\tag{C1.5.1}$$

where the parameters for matrices  $A$  and  $B$  are obtained using (C1.7),  $u$  – the (deterministic) input,  $w$  – the white process noise and  $v$  – the white measurement noise. Knowing that  $E(w)=E(v)=0$ ,  $E(ww^T)=Q$  ( $Q$  – process noise covariance),  $E(vv^T)=R$  ( $R$  – measurement noise covariance) and  $E(wv^T)=N$ , we can construct a state estimate  $\hat{x}$  that minimizes the steady-state error covariance according to:

$$P = \lim_{t \rightarrow \infty} E\left\{\{x - \hat{x}\}\{x - \hat{x}\}^T\right\}\tag{C1.5.2}$$

In this context, the equations of optimal Kalman filter-based solution are:

$$\begin{aligned}\dot{\hat{x}} &= A \hat{x} + B u + L(y - C \hat{x} - D u), \\ \begin{bmatrix} \hat{y} \\ \hat{x} \end{bmatrix} &= \begin{bmatrix} C \\ I \end{bmatrix} \hat{x} + \begin{bmatrix} D \\ 0 \end{bmatrix} u.\end{aligned}\tag{C1.5.3}$$

By solving an algebraic Riccati equation we can determine the filter gain  $L$  according to:

$$L = (PC^T + \bar{N})\bar{R}^{-1},\tag{C1.5.4}$$

where  $\bar{R} = R + HN + N^T H^T + HQH^T$  and  $\bar{N} = G(QH^T + N)$ . The KFO block diagram is presented in Fig. C1.5.1.

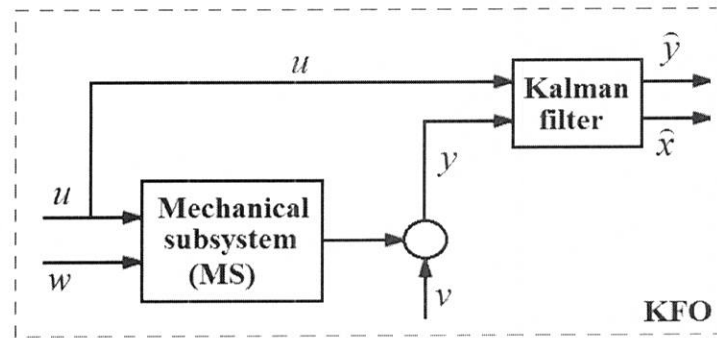


Fig. C1.5.1. KFO block diagram.

## C2. State observers for mechatronic systems with rigid and flexible transmission

The development of suitable algorithms to perform the estimation has been the focus of many researchers' attention and for this purpose, in order to estimate state variables from the available measurements, several techniques have been developed and introduced (Brown and Hwang, 1996; Aghannan and Rouchon, 2003; Marx et al., 2007; Lendek et al., 2008; Spurgeon, 2008; Magnis and Petit, 2016). In this context, the paper proposes a controller that can be considered as composed of three subsystems: a subsystem that provides the desired output and from the reference value a feed-forward signal, an observer and a feedback derived from the estimated states. The interesting structure of the controller allows it to be applied for a wide range of design methods. The controller structure and the forms of the equations are exactly the same for systems



with one input and one output as well as for systems with multiple inputs and outputs. The same controller structure can be obtained by employing many other design techniques. The defining feature of a state feedback controller and an observer have is the complexity of the controlled system that determines controller's complexity. As such a system model is actually contained by the controller. Thereby the internal model principle that prescribes that an internal model of the controlled system should be contained in the controller is in this research report exemplified.

The laboratory application - Model 220 Industrial Plant Emulator (MIPE220), Fig C2.1 - considered in this chapter is a complex, nonlinear equipment, ideal for studying automatic control solutions dedicated to the control of modern industrial equipment, such as : drive shafts, conveyors, rotary tables, machine tools and automated assembly machines (ECP, 2010; Szedlak-Stinean et al., 2016; Szedlak-Stinean et al., 2017). The dynamic equations that describe the mechatronics system in case of rigid (a) and flexible (b) drive dynamics, considering  $\theta_1$  as the process output are:

$$\begin{aligned} a) \quad & J_{dr} \ddot{\theta}_1 + (c_1 + c_2 g^{-2}) \dot{\theta}_1 = T_D, \\ b) \quad & J_{dr} \ddot{\theta}_1 + (c_1 + c_{12} g^{-2}) \dot{\theta}_1 - c_{12} g^{-1} \dot{\theta}_2 + k(g^{-2} \theta_1 - g^{-1} \theta_2) = T_D, \\ & J_l \ddot{\theta}_2 + (c_2 + c_{12}) \dot{\theta}_2 - c_{12} g^{-1} \dot{\theta}_1 + k(\theta_2 - g^{-1} \theta_1) = 0. \end{aligned} \quad (C2.1)$$

with  $J_{dr}$ ,  $J_d$ ,  $J_l$ ,  $J_p$ ,  $g$  and  $g'$  expressed as:

$$\begin{aligned} J_{dr} &= J_d + J_p g'^{-2} + J_l g^{-2}, J_d = J_{ldr} + J_{wdr}, \\ J_p &= J_{pdr} + J_{pld} + J_{backlash}, J_l = J_{lld} + J_{wld}, \\ g &= 6n_{pd} / n_{pl}, g' = n_{pd} / 12. \end{aligned} \quad (C2.2)$$

where  $J_{dr}$  – total inertia reflected to the drive disk,  $J_p$ ,  $J_d$ ,  $J_l$  – pulley, drive disk and load disk inertia,  $c_1$ ,  $c_2$  – the drive and load friction,  $g$ ,  $g'$  – drive gear and partial gear system ratio,  $\theta_1$ ,  $\theta_2$ ,  $\theta_p$  – drive disk, load disk and idler pulleys positions where  $\theta_1 = g\theta_2$  or  $\theta_1 = g'\theta_p$ .

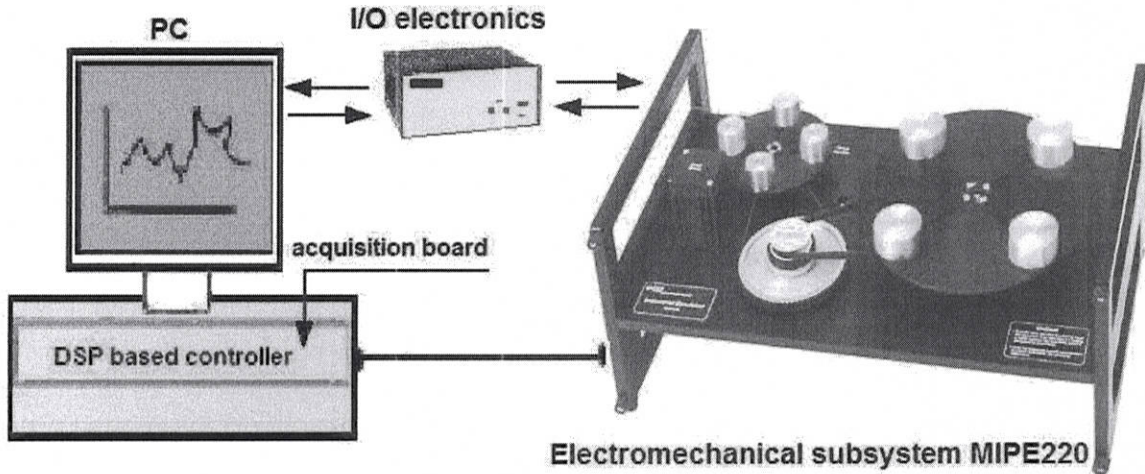


Fig. C2.1. MIPE220 laboratory equipment.

#### Rigid Drive Dynamics

The first principle equations that describe the system in case of rigid drive dynamics are (ECP, 2010; Szedlak-Stinean et al., 2016):

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = \frac{-(c_1 + c_2 g^{-2})x_2}{J_d + J_p g'^{-2} + J_l g^{-2}} + \frac{T_D}{J_d + J_p g'^{-2} + J_l g^{-2}}, \\ v = x_1. \end{cases} \quad (C2.3)$$

The state-space MM (SS-MM) of MIPE220 with rigid drive dynamics is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & \frac{-(c_1 + c_2 g^{-2})}{J_d + J_p g^{r-2} + J_l g^2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} T_D, \quad (C2.4)$$

$$y = [1 \ 0] [x_1 \ x_2]^T.$$

where  $T_D$  is the drive torque ( $T_D = u$ ),  $x = [x_1 \ x_2]^T = [\theta_1 \ d\theta_1/dt]^T$  is the state vector ( $T$  indicates matrix transposition) and  $y$  is the output. Considering zero initial conditions, the application of the Laplace transform to (C2.1a)) leads to the following transfer function (t.f.):

$$\frac{\theta_1(s)}{T_D(s)} = \frac{1/(J_d + J_p g^{r-2} + J_l g^2)}{s[s + (c_1 + c_2 g^{-2})/(J_d + J_p g^{r-2} + J_l g^2)]}, \quad (C2.5)$$

### Flexible Drive Dynamics

The first principle equations that describe the system in case of flexible drive dynamics are (ECP, 2010):

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = \frac{-(kg^{-2})x_1}{J_{dr}} + \frac{-(c_1 + c_{12}g^{-2})x_2}{J_{dr}} + \frac{kg^{-1}}{J_{dr}} + \frac{c_{12}g^{-1}}{J_{dr}}, \\ \dot{x}_3 = x_4, \\ \dot{x}_4 = \frac{(kg^{-1})x_1}{J_l} + \frac{(c_{12}g^{-1})x_2}{J_l} + \frac{(-k)x_3}{J_l} + \frac{-(c_2 + c_{12})x_4}{J_l}, \\ y = x_1. \end{cases} \quad (C2.6)$$

The SS-MM of MIPE220 with flexible drive dynamics is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-(kg^{-2})}{J_{dr}} & \frac{-(c_1 + c_{12}g^{-2})}{J_{dr}} & \frac{kg^{-1}}{J_{dr}} & \frac{c_{12}g^{-1}}{J_{dr}} \\ 0 & 0 & 0 & 1 \\ \frac{kg^{-1}}{J_l} & \frac{c_{12}g^{-1}}{J_l} & \frac{-k}{J_l} & \frac{-(c_2 + c_{12})}{J_l} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{J_d + J_p g^{r-2} + J_l g^2} \\ 0 \\ 0 \end{bmatrix} T_D, \quad (C2.7)$$

$$y = [1 \ 0 \ 0 \ 0] [x_1 \ x_2 \ x_3 \ x_4]^T.$$

where  $T_D$  is the drive torque ( $T_D = u$ ,  $u$  is the input),  $x = [x_1 \ x_2 \ x_3 \ x_4]^T = [\theta_1 \ d\theta_1/dt \ \theta_2 \ d\theta_2/dt]^T$  is the state vector and  $y$  is the output. The following t.f. is attached to (C2.7):

$$\frac{\theta_1(s)}{T_D(s)} = \frac{J_l s^2 + (c_2 + c_{12})s + k}{d_4 s^4 + d_3 s^3 + d_2 s^2 + d_1 s}, \quad (C2.8)$$

where  $d_4 = J_{dr} J_l$ ,  $d_3 = J_{dr}(c_2 + c_{12}) + J_l(c_1 + c_{12}/g^2)$ ,  $d_2 = J_{dr}k + J_l k/g^2 + c_1 c_2 + c_1 c_{12} + c_{12} c_2/g^2$ ,  $d_1 = c_1 k + c_2 k/g^2$ .

### C2.1. State feedback and observer-based controller design

In cases where the process states are not accessible for measurements or are only partially accessible for measurements and if the process is observable, then it is possible to estimate its states. For this purpose, state estimators or state observers are utilized. The observability test of the linearized SS-MMs (C2.4) and (C2.7) can be done using the matrix:

$$Q_o = [C^T \ C^T A \ C^T A^2 \ C^T A^3 \ C^T A^4 \dots]^T. \quad (C2.1.1)$$

The starting point in order to specify the relations that describe the functioning of a state

observer, is the SS-MM corresponding to the process, assumed known, with the form:

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{A} \mathbf{x} + \mathbf{B} u, \\ y &= \mathbf{C} \mathbf{x}.\end{aligned}\tag{C2.1.2}$$

The variable that is the target of the control process is the output. Firstly, all components of the state vector are assumed as measured. The feedback is constrained to be linear, so it can be considered as (Åström and Murray, 2009):

$$u = -\mathbf{K} \mathbf{x} + K_{ref} r\tag{C2.1.3}$$

where  $r$  is the reference input,  $K_{ref}$  is the feed-forward gain and  $\mathbf{K}$  is the state feedback gain matrix. The state feedback gain matrix of MIPE220 with rigid (a) and flexible (b) drive dynamics are:

$$\begin{aligned}a) \quad \mathbf{K} &= [k_{c1} \quad k_{c2}], \\ b) \quad \mathbf{K} &= [k_{c1} \quad k_{c2} \quad k_{c3} \quad k_{c4}].\end{aligned}\tag{C2.1.4}$$

The pole placement method is applied to compute  $\mathbf{K}$  using three sets of imposed poles, each for three specific load disk inertia values, i.e.,  $J_{l1}$ ,  $J_{l2}$ ,  $J_{l3}$ . The closed loop system obtained when the feedback (C2.1.3) is applied to the system (C2.1.2) is given by:

$$\dot{\mathbf{x}} = (\mathbf{A} - \mathbf{B} \mathbf{K}) \mathbf{x} + \mathbf{B} K_{ref} r.\tag{C2.1.5}$$

The SS-MM corresponding to the state observer has the same structure as the process (C2.1.2) and is completed with a correction relation based on the output error  $\tilde{y} = y - \hat{y}$ . Consequently, the MM is rewritten in the form (Åström and Murray, 2009):

$$\begin{aligned}\dot{\hat{\mathbf{x}}} &= \mathbf{A} \hat{\mathbf{x}} + \mathbf{B} u + \mathbf{L}(y - \mathbf{C} \hat{\mathbf{x}}) = (\mathbf{A} - \mathbf{L} \mathbf{C}) \hat{\mathbf{x}} + \mathbf{B} u + \mathbf{L} y, \\ \hat{y} &= \mathbf{C} \hat{\mathbf{x}},\end{aligned}\tag{C2.1.6}$$

where  $\mathbf{L}$  is the observer gain. The parameters of the observer gain for MIPE220 with rigid (a) and flexible (b) drive dynamics are:

$$\begin{aligned}a) \quad \mathbf{L} &= [l_1 \quad l_2]^T, \\ b) \quad \mathbf{L} &= [l_1 \quad l_2 \quad l_3 \quad l_4]^T.\end{aligned}\tag{C2.1.7}$$

In order to analyze the observer, the state estimation error is defined as  $\tilde{\mathbf{x}} = \mathbf{x} - \hat{\mathbf{x}}$ . Differentiating and replacing the expressions of  $\dot{\hat{\mathbf{x}}}$  and  $\dot{\mathbf{x}}$  leads to  $\dot{\tilde{\mathbf{x}}} = (\mathbf{A} - \mathbf{L} \mathbf{C}) \tilde{\mathbf{x}}$ . The error  $\tilde{\mathbf{x}}$  will go to zero if the matrix  $\mathbf{L}$  is chosen such that the matrix  $(\mathbf{A} - \mathbf{L} \mathbf{C})$  has eigenvalues / poles with negative real parts. The appropriate selection of the eigenvalues / poles determines the convergence rate (Åström and Murray, 2009). Taking this into account, the design of the state observer involves solving a poles placement problem and also calculating the parameters of the observer gain. The starting point in designing the state observer is the expression of the characteristic polynomial:

$$\Delta_{ob}(s) = \det(s \mathbf{I} - \mathbf{A} + \mathbf{L} \mathbf{C}) = s^n + \alpha_{n-1} s^{n-1} + \dots + \alpha_1 s + \alpha_0.\tag{C2.1.8}$$

By allocating the poles of the observer, the characteristic polynomial  $\Delta_{ob}(s)$  is expressed as:

$$\Delta_{ob}(s) = \prod (s - p_{oi}) = s^n + \beta_{n-1} s^{n-1} + \dots + \beta_0\tag{C2.1.9}$$

Because both the system (C2.1.2) and the observer (C2.1.6) have the same order  $n$ , the order of the closed loop system is  $2n$ . In order to obtain the state feedback observer, the design of the observer as well as the design of the state feedback can be realized separately. The closed-loop system is defined as:

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\tilde{\mathbf{x}}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{B} \mathbf{K} & \mathbf{B} \mathbf{K} \\ 0 & \mathbf{A} - \mathbf{L} \mathbf{C} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \tilde{\mathbf{x}} \end{bmatrix} + \begin{bmatrix} \mathbf{B} K_{ref} \\ 0 \end{bmatrix} r\tag{C2.1.10}$$



Due to the fact that the matrix on the right side is block diagonal, the characteristic polynomial of the closed-loop system has the form:

$$\Delta_{\text{sys}}(s) = \det(sI - A + BK) \det(sI - A + LC). \quad (\text{C2.1.11})$$

This property is called the separation principle (Åström and Murray, 2009). A schematic diagram of the controller is illustrated in Fig. C2.1.1. It can be observed that the controller includes a dynamic model of the plant, thus respecting the internal model principle. It can also be noticed that the observer determines the dynamics of the controller. As such, the controller can be regarded as a dynamical system having  $y$  as input and  $u$  as output:

$$\begin{aligned} \dot{\hat{x}} &= (A - BK - LC)\hat{x} + L y, \\ u &= -K \hat{x} + K_{\text{ref}} r. \end{aligned} \quad (\text{C2.1.12})$$

The t.f. of the controller has the form:

$$H_c(s) = K [sI - A + BK + LC]^{-1} L. \quad (\text{C2.1.13})$$

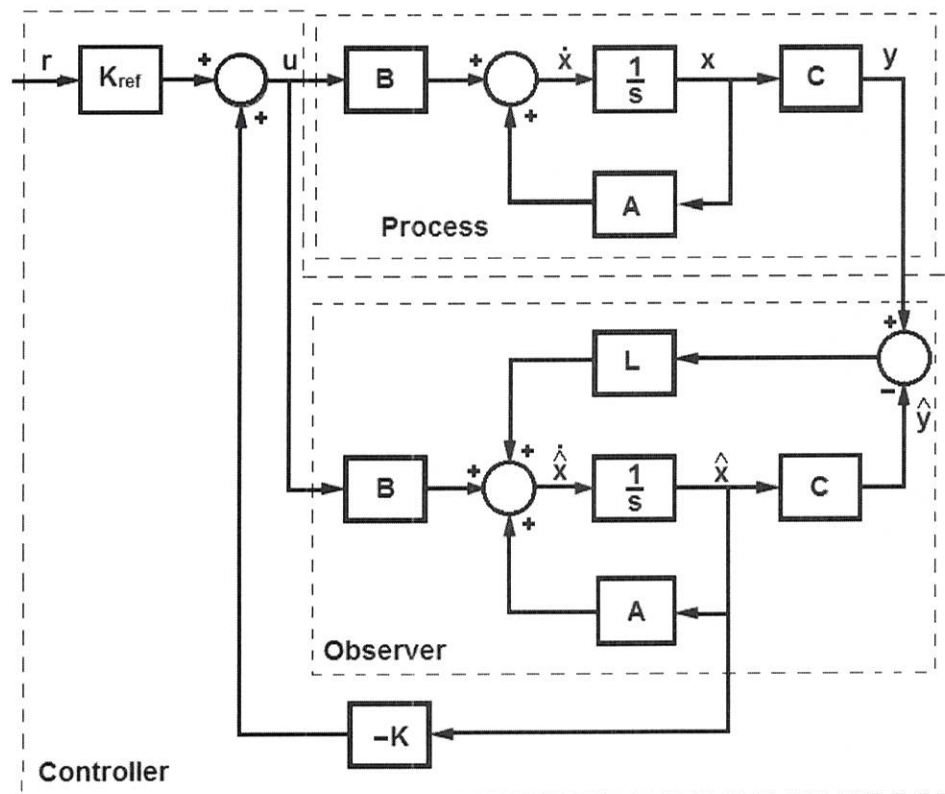


Fig. C2.1.1. Schematic diagram of an observer-based controller.

#### D. Bibliography

- J. Ackermann, *Der Entwurf linearer Regelungssysteme im Zustandsraum*. Regelungstechnik, vol. 20, pp. 297-300, 1972.
- N. Aghannan și P. Rouchon, "An Intrinsic Observer for a Class of Lagrangian Systems," *IEEE Trans. Aut. Control*, vol. 48, no. 6, 2003.
- K. J. Åström și R.M. Murray, 2009. *Feedback Systems. An introduction for scientists and engineers*. Princeton, New Jersey, Princeton University Press.
- G.-D. Andreescu, *Estimatoare în sisteme de conducere a acționărilor electrice: aplicații la mașini sincrone cu magneți permanenți*. Timisoara: Editura Orizonturi Universitare, 1999.
- W.A. Apaza-Perez, J.A. Moreno și L.M. Fridman, "Global sliding mode observers for a class of

- mechanical systems with disturbances," in *Proc. 10th IFAC Symposium on Nonlinear Control Systems (NOLCOS 2016)*, Monterey, CA, USA, pp. 1–6, 2016.
- S. Baldursson, *BLDC motor modelling and control – A Matlab/Simulink implementation*. M.Sc. Thesis, Inst. Energi Miljö, Göteborg, Sweden, 2005.
- P. Bergsten, R. Palm și D. Driankov, "Observers for Takagi–Sugeno fuzzy systems," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 32, no. 1, pp. 114–121, 2002.
- S. Beyhan, Z. Lendek, M. Alci, R. Babuska, "Takagi-Sugeno fuzzy observer and extended Kalman filter for adaptive payload estimation," in *Proc. 2013 Asian Control Conference (ASCC 2013)*, Istanbul, Turkey, pp. 1–6, 2013.
- S. Biagiola și J. Solsona, "State estimation in batch processes using a nonlinear observer," *Math. Comput. Model.*, vol. 44, no. 11–12, pp. 1009–1024, 2006.
- R. H. Bishop, *The Mechatronics Handbook*. Boca Raton, FL: CRC Press, 2<sup>nd</sup> edition, 2007.
- D. Boulfelfel, R. M. Rangayyan, L. J. Hahn și R. Kloiber, "Three-dimensional restoration of single photon emission computed tomography images," *IEEE Trans. Nucl. Sci.*, vol. 41, no. 5, pp. 1746–1754, Oct. 1994.
- R. G. Brown și P.Y.C. Hwang, *Introduction to Random Signals and Applied Kalman Filtering*. New York: John Wiley & Sons, 3<sup>rd</sup> edition, 1996.
- R. C. Dorf și R. H. Bishop, *Modern Control Systems*. Pearson Educational International, Upper Saddle River, 2005.
- C.-A. Dragos, *Soluții moderne de reglare bazate pe model cu aplicații în sisteme mecatronice*, Teză de doctorat, Universitatea Politehnică Timisoara, Seria 12: Ingineria Sistemelor, no. 2, 2011.
- I.-M. Dragu, *Soluții de reglare pentru vehicule cu tracțiune electrică (hibridă)*. B.Sc. Thesis, Politehnica Univ. Timisoara, Timisoara, Romania, 2009.
- ECP, 2010. *Industrial Emulator/Servo Trainer Model 220 System, Testbed for Practical Control Training*. Bell Canyon, CA: Educational Control Products.
- M. Essabre, J. Soulami, A. El Assoudi, E. Elyaagoubi și E. El Bouatmani, "Fuzzy Observer Design for a Class of Takagi-Sugeno Descriptor Systems," *Contemporary Engineering Sciences*, vol. 7, no. 4, 149–164, 2014.
- V. Estrada-Manzo, Z. Lendek și T. M. Guerra, "Discrete-time Takagi-Sugeno descriptor models: observer design," in *Proc. 19<sup>th</sup> World Congress, IFAC, Cape Town, South Africa*, pp. 7965–7969, August 2014.
- B. Friedland, *Advanced Control System Design*. Prentice Hall, Englewood Cliffs, 1996.
- Z. Gajic și M. Lelic, *Modern Control Systems Engineering*. Prentice Hall International, London, 1996.
- R. A. Garcia, M. I. Troparevsky și J. L. Mancilla Aguilar, "An observer for nonlinear noisy systems," *Latin American Applied Research*, vol. 30, no. 2, pp. 87–92, 2000.
- M. Ghanes, J. De Leon și J. Barbot, "Observer design for nonlinear systems under unknown time-varying delays," *IEEE Trans. Autom. Control*, vol. 58, no. 6, pp. 1529–1534, 2013.
- M. Habibullah și D.D.-C. Lu, "A speed-sensorless FS-PTC of induction motors using extended Kalman filters," *IEEE Trans. Ind. Electron.*, vol. 62, no. 11, pp. 6765–6778, 2015.
- H. Hamdi, M. Rodrigues, Ch. Mechmech și N. Benhadj Braiek, "Observer based Fault Tolerant Control for Takagi-Sugeno Nonlinear Descriptor systems," in *Proc. International Conference on Control, Engineering and Information Technology (CEIT 2013)*, vol. 1, pp. 52–57, 2013.
- D. Hanselman, *Brushless Permanent Magnet Motor Design*. New York: McGraw-Hill, 2006.
- D. Hansson, *Study and test of a BLDC motor drive systems function and operation for a pump application*. M.Sc. Thesis, Instit. Elteknik, Chalmers Tekniska Hogskola, Chalmers, Sweden, 2004.
- P. Hehenberger, R. Naderer, C. Schuler și K. Zeman, "Conceptual design of mechatronic systems as a recurring element of innovation processes," in *Proc. 4<sup>th</sup> IFAC Symposium on Mechatronic System (MECHATRONICS 2006)*, Heidelberg, Germany, pp. 342–347, 2006.

- C.-P. Huang, "Stability analysis of discrete singular fuzzy systems," *Fuzzy Sets Syst.*, vol. 151, no. 1, pp. 155–165, 2005.
- D. Ichalal, B. Marx, J. Ragot, D. Maquin, "Design of observers for Takagi-Sugeno discrete-time systems with unmeasurable premise variables," in *5<sup>th</sup> Workshop on Advanced Control and Diagnosis (ACD 2007)*, Grenoble, France, November 2007.
- K. Ilhem, H. Rafika, S. Bel Hadj Ali și A. M. Naceur, "Observer Design for Takagi-Sugeno Descriptor System with Lipschitz Constraints," *Int. J. Instrum. Control Syst.*, vol. 2, no.2, April 2012.
- R. Isermann, *Mechatronic Systems. Fundamentals*. Berlin, Heidelberg, New York: Springer-Verlag, 2005.
- J. C. Kinsey și L. L. Whitcomb, "Model-based nonlinear observers for underwater vehicle navigation: theory and preliminary experiments," in *Proc. IEEE International Conference on Robotics and Automation (ICRA 2007)*, Rome, Italy, pp. 4251–4256, 2007.
- C. Koch, O. Radler, A. Spröwitz, T. Ströhlä și V. Zöppig, *Project course Design mechatronic systems*, 2006.
- H.K. Lam, H. Li și H. Liu, "Stability analysis and control synthesis for fuzzy-observer-based controller of nonlinear systems: a fuzzy model-based control approach," *IET Control Theory Appl.*, vol. 7, no. 5, pp. 663–672, 2013.
- D. H. Lee, Y. H. Joo și M. H. Tak, "Linear matrix inequality approach to local stability analysis of discrete-time Takagi–Sugeno fuzzy systems," *IET Control Theory Appl.*, vol. 7, no. 9, pp. 1309–1318, 2013.
- Z. Lendek, R. Babuska și B. De Schutter, "Distributed Kalman filtering for cascaded systems," *Eng. Appl. Artif. Intell.*, vol. 21, no. 3, pp. 457–469, 2008.
- Z. Lendek, T. M. Guerra, R. Babuska și B. De Schutter, *Stability analysis and nonlinear observer design using Takagi-Sugeno fuzzy models*. Springer, 2010.
- F. Li, P. Shi, L. Wu și H. R. Karimi, "D-Stability for Discrete-Time T-S Fuzzy Descriptor Systems with Multiple Delays," in *Proc. American Control Conference (ACC 2014)*, Portland, Oregon, USA, 2014.
- D. G. Luenberger, "Observers for multivariable systems," *IEEE Trans. Automat. Contr.*, vol. 11, no. 2, pp. 190–197, April 1966.
- L. Magnis și N. Petit, "Angular velocity nonlinear observer from single vector measurements," *IEEE Trans. Autom. Control*, vol. 61, no. 9, pp. 2473–2483, 2016.
- G. F. Marshall and G. E. Stutz, *Handbook of Optical and Laser Scanning*, 2<sup>nd</sup> ed. Boca Raton, FL: CRC Press, 2012.
- B. Marx, D. Koenig și J. Ragot, "Design of observers for Takagi-Sugeno descriptor systems with unknown inputs and application to fault diagnosis," *IET Control Theory Appl.*, vol. 5, 1487–1495, 2007.
- R. Măgureanu and N. Vasile, *Servomotoare fără perii tip sincron*. Bucharest: Editura Tehnica, 1990.
- P. Mercorelli, "A two-stage sliding-mode high-gain observer to reduce uncertainties and disturbances effects for sensorless control in automotive applications," *IEEE Trans. Ind. Electron.*, vol. 62, no. 9, pp. 5929–5940, 2015.
- Ph. Mouyon, "Tools for nonlinear observer design," in *Proc. IEEE International Symposium on Diagnostics and Drivers (SDEMPED 1997)*, Carry-Le-Rouet, France, 1997.
- S. A. Nasar and I. Boldea, *Electric Drives*. 2nd ed. New York: CRC Press, Taylor & Francis, 2005.
- I. Pabst, "An approach for reliability estimation and control of mechatronic systems," in *Proc. 4<sup>th</sup> IFAC Symposium on Mechatronic Systems (MECHATRONICS 2006)*, Heidelberg, Germany, pp. 831–836, 2006.
- R. Palm și P. Bergsten, "Sliding mode observer for a Takagi–Sugeno fuzzy system," in *Proc. 9th*



- IEEE International Conference on Fuzzy Systems (FUZZ IEEE 2000).*, San Antonio, TX, USA, vol. 2, pp. 665–670, 2000.
- S. Preitl, R.-E. Precup și Zs. Preitl, *Structuri și algoritmi pentru conducerea automată a proceselor*. vol. 1 și 2, Editura Orizonturi Universitare, Timisoara, 2009.
- S. Preitl și R.-E. Precup, *Tehnici de proiectare a structurilor de reglare automată. Aplicații*. Editura Orizonturi Universitare, Timisoara, 2008.
- D. Simon, *Optimal State Estimation: Kalman, H Infinity, and Nonlinear Approaches*. Wiley-Interscience, 2006.
- M. Soroush, "Nonlinear state-observer design with application to reactors," *Chem. Eng. Sci.*, vol. 52, no. 3, pp. 387–404, 1997.
- J. Soulami, A. El Assoudi, M. Essabre, M. Habibi și E. El Yaagoubi, "Observer Design for a Class of Class of Nonlinear Descriptor Systems: A Takagi-Sugeno Approach with Unmeasurable Premise Variables," *J. Control Sci. Eng.*, vol. 2015, pp. 10, 2015.
- Y. Shtessel, C. Edwards, L. Fridman, and A. Levant, *Sliding Mode Control and Observation*. Berlin, Heidelberg: Springer, 2014.
- S.K. Spurgeon, "Sliding mode observers: A survey," *Int. J. Syst. Sci.*, vol. 39, no. 8, pp. 751–764, 2008.
- Stefani. Shahian, Savant și Hostetter, *Design of Feedback Systems*. Oxford University Press, New York, 2002.
- A.-I. Stinean, *Contribuții la dezvoltarea unor soluții de reglare dedicate sistemelor de acționare electrică cu parametri variabili și cu intrări variabile în timp*. Timisoara: Editura Politehnica, 2014.
- A.-I. Szedlak-Stinean, R.-E. Precup, S. Preitl, E. M. Petriu, C.-A. Bojan-Dragos, 2016. State feedback control solutions for a mechatronics system with variable moment of inertia. In *Proc. 13<sup>th</sup> International Conference on Informatics in Control, Automation and Robotics*, Lisbon, Portugal, 458-465.
- A.-I. Szedlak-Stinean, R.-E. Precup, E. M. Petriu, 2017. Fuzzy and 2-DOF controllers for processes with a discontinuously variable parameter. In *Proc. 14<sup>th</sup> International Conference on Informatics in Control, Automation and Robotics*, Madrid, Spain, 431-438.
- T. Takagi și M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. SMC-15, no. 1, pp. 116–132, January/February 1985.
- K. Tanaka, T. Ikeda și H.O. Wang, "Fuzzy regulators and fuzzy observers: relaxed stability conditions and LMI-based designs," *IEEE Trans. Fuzzy Syst.*, vol. 6, no. 2, pp. 250–265, 1998.
- K. Tanaka și H. O. Wang, *Fuzzy control systems design and analysis: A Linear Matrix Inequality Approach*. John Wiley & Sons, New York, USA, 2001.
- T. Taniguchi, K. Tanaka, H. Ohtake și H. Wang, "Model construction, rule reduction, and robust compensation for generalized form of Takagi-Sugeno fuzzy systems," *IEEE Trans. Fuzzy Syst.*, vol. 9, no.4, pp. 525-538, August 2001.
- T. Taniguchi, K. Tanaka și H.O. Wang, "Fuzzy Descriptor Systems and Nonlinear Model Following Control," *IEEE Trans. Fuzzy Syst.*, vol. 8, no.4, pp. 442-452, August 2000.
- F. E. Thau, "Observing the state of nonlinear dynamic systems," *Int. J. Contr.*, vol. 17, pp 471-479, 1973.
- Z. Wang, Y. Shen, X. Zhan și Q. Wang, "Observer design for discrete-time descriptor systems: An LMI approach," *Syst. Control Lett.*, vol 61, no. 6, pp. 683-687, 2012.
- N. Wiener, *Extrapolation, Interpolation, and Smoothing of Stationary Time Series*. New York: Wiley, 1949.
- P. Yedamale, *Brushless DC (BLDC) Motor Fundamentals. Application Note 885*. Chandler, AZ, USA: Microchip Technology Inc., 2003.

## **E. Published and ongoing works**

### **- papers published in conference volumes indexed in international databases**

- [1] A.-I. Szedlak-Stinean, R.-E. Precup and R.-C. David, "Speed and acceleration control of BLDC drives using different types of observers," in *Proc. IEEE 13<sup>th</sup> International Symposium on Applied Computational Intelligence and Informatics (SACI 2019)*, Timisoara, Romania, 2019, pp. 229-236.
- [2] A.-I. Szedlak-Stinean, R.-E. Precup and R.-C. David, "State observers for mechatronics systems with rigid and flexible drive dynamics," in *Proc. IEEE 16<sup>th</sup> International Conference on Informatics in Control, Automation and Robotics (ICINCO 2019)*, Prague, Czech Republic, 2019, pp. 387-394.
- [3] C.-A. Bojan-Dragos, E.-L. Hedrea, R.-E. Precup, A.-I. Szedlak-Stinean and R.-C. Roman, "MIMO fuzzy control solutions for the level control of vertical two tank systems," in *Proc. IEEE 16<sup>th</sup> International Conference on Informatics in Control, Automation and Robotics (ICINCO 2019)*, Prague, Czech Republic, 2019, pp. 810-817.
- [4] R.-C. Roman, R.-E. Precup, C.-A. Bojan-Dragos and A.-I. Szedlak-Stinean, "Combined model-free adaptive control with fuzzy component by virtual reference feedback tuning for tower crane systems," in *Proc. Information Technology and Quantitative Management (ITQM 2019)*, Granada, Spain, 2019, pp. 8.

### **- works in progress (ongoing works)**

- [5] R.-E. Precup, E.-I. Voisan, E.-M. Petriu, R.-C. David and A.-I. Szedlak-Stinean, "Grey wolf optimizer-based approaches to path planning and fuzzy logic-based tracking control for mobile robots," 2019, 8 pp.
- [6] R.-C. David, R.-E. Precup, S. Preitl, A.-I. Szedlak-Stinean, E.-M. Petriu and R.-C. Roman, "Whale optimization algorithm-based tuning of low cost fuzzy controllers with reduced process parametric sensitivity," 2019, 6 pp.

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