Intermediate scientific report of the contract PN-III-P1-1.1-PD-2016-0331
regarding the implementation of the project during december 2021

A. Overall summary of the project

The research team and the roles of the members who carried out research activities within the project *"Nonlinear observers-based control structures applied to mechatronics systems"*, financing contract no. 164/10.10.2018, with the deposit code PN-III-P1-1.1-PD-2016-0331, https://szedlak-stinean.wixsite.com/nobsmecs, is the one nominated in the funding application: Lect. Dr. Ing. Alexandra-Iulia Szedlak-Stînean, the project leader and Prof. Dr. Ing. Radu-Emil Precup, the mentor.

Due to the suspension of the project starting from 01.10.2019 and until 30.11.2021, the reason for the interruption being the entry into maternity leave and then the entry into child-raising leave, the *main objective* pursued during the contract for the month of December 2021 was the same as in *Stage 1 – The development of the theoretical framework which allows the implementation of the modern control solutions*, then following that this knowledge will help to implement, test, verify and validate the solutions developed through simulations and through experiments, in other words the development of activities related to *Stages 2, 6 and 7*. To achieve the objective, the following activities were carried out:

1.1. Obtaining current theoretical knowledge regarding observers. To accomplish this activity the analysis of current theoretical research – state-of-the-art on theoretical research – and the possibilities of improving modern control solutions were considered. Details are presented in the study from section B.

1.2. Studying the practical applications of various types of observers. To accomplish this activity, the realization and classification of observers was studied with focus on mechatronic systems. Details are also presented in the study from section B.

For the **year 2021**, more precisely for the month of december, the present intermediate scientific report (research report) is the only deliverable of the project.

Remarks:

1. The bibliographic study related to section B is presented at the end of the intermediate scientific report, more precisely in section C.

2. In this research report, annotations in english may appear in this. Each time, however, the necessary clarifications are made to make the interpretations as clear as possible.

3. The results obtained are also mentioned on the project web page, https://szedlak-stinean.wixsite.com/nobsmecs, where all the information related to the project will be included.

B. The development of the theoretical framework which allows the implementation of the modern control solutions. Study on acquiring current theoretical knowledge regarding observers and analyzing the practical applications of various types of observers

The research conducted during 2010-2021 concerns the analysis, synthesis and modeling of mechatronics systems based on electric drives. Modern control solutions are reviewed and developed dedicated towards controlling the speed and position of electric drives with variable parameters. The control strategies of servo-systems should provide very good dynamic performance and steady-state conditions for the developed control systems. In this sense, the main research was oriented towards the development of new control solutions dedicated to speed and position control of three mechatronics applications:

1. The electric drive system which wraps a strip with constant linear velocity on a drum - electric drive system with variable parameters.

2. The electric drive system with time-varying inputs with a brushless motor.
3. The laboratory equipment - Model 220 Industrial Plant Emulator.

Mechatronics systems have experienced a rapid and complex multidisciplinary development as a result of advances in various application areas such as (Bishop, 2007; Gutiérrez-Carvajal et al., 2016; Isermann, 2005): expert systems, automotive engineering, robotics and automation, systems structural dynamics, control systems, servomechanics, numerical computing systems based on highly integrated microelectronics, consumer products, medical imaging systems, mobile applications, integrated and computer-aided manufacturing systems, transportation systems and vehicles. Mechatronics systems are successfully used in many industrial and non-industrial applications because of their simple and robust structure. By constantly evolving, these systems exhibit increasing performance ensuring functional and applicative versatility, intelligence and flexibility. These features are also provided by the control system which has to be able to adapt to external conditions at all times and to provide the necessary informations to the hierarchical control (Isermann, 2005; Koch et al., 2006; Hehenberger et al., 2006; Pabst, 2006; Bishop, 2007; Dragos, 2011). Mechatronic applications incorporate:

- Mechanical subsystem (the controlled process);
- Electronic interface subsystems (the execution and measuring elements);
- Control subsystem.

The degree of complexity of the structure and of the control subsystem of a mechatronic application differs from one application to another and may include relatively simple and advanced control structures. The following points of view may be considered in the development of mechatronic applications (Dragos, 2011):

- accepting a simplified system representation;
- establishing the operating performance;
- control of specific measurements, including state estimation;
- generating dynamic behavior in special situations;
- development of advanced algorithms;
- detecting and diagnosing operating errors.

The design of control systems is important because of the very good performance specifications imposed in various fields. The specifications become more restrictive if several operating conditions of the processes are involved as the variable parameters (for example, in the case of speed and position control besides the variable moment of inertia situations, the variable reference input, correlated with various particular operating conditions, additional operating conditions may occur, such as oscillations that apear on the mechanical side). These problems can be solved in two ways: by developing advanced nonlinear models or by implementing a robust control based on state observers technique. The implementation of the second solution in servo-systems leads to high system performances. In this context, the development of advanced control solutions is a relatively difficult problem which requires a detailed preliminary study of the process, particularly the parameters variability and of its structural properties. As a continuation of previous research, in this project (PD 164/10.10.2018) the approach oriented on the analysis, synthesis, modeling and development of modern control solutions, such as nonlinear observers dedicated to certain mechatronics applications is proposed. Two types of observers can be used: linear and nonlinear. From the category of **linear observers**, the most common are **full-order observers** known as the Luenberger type observers and **reduced-order observers**. Among the nonlinear ones we mention: **Kalman-extended filters, sliding mode observers, fuzzy** and **neural network observers** (Luenberger, 1966; Ackermann, 1972; Friedland, 1996; Gajic şi Lelic, 1996; Stefani et al., 2002; Preitl et al., 2009).

B1. Control structures based on linear observers intended for mechatronic applications.

The topic of designing observers for linear systems was proposed and solved for the first time by Luenberger (Luenberger, 1966). The linear observers that are discussed in this subchapter are implemented on two mechatronic applications: 1. the electric drive system with time-varying inputs with BLDC motor and 2. the laboratory equipment – Model 220 Industrial Plant Emulator. First time, the observers applied to the electric drive system with time-varying inputs with BLDC motor will be shown and will be presented in sections B1.1, B1.2, B1.3 and B1.4, followed by the observers applied to the laboratory equipment which will be presented in section B1.5.

I. Electric drive system with time-varying inputs with BLDC motor

The electric drive systems with Brushless Direct Current (BLDC) motors (BLDC drives) have experienced a fast and complex multidisciplinary development as a result of advances in various fields such as (Yedamale, 2003; Hansson, 2004; Baldursson, 2005; Nasar and Boldea, 2005): power electronics, modern automated control techniques, powering electric vehicles (e.g., cars, electric bicycles, electric scooters), numerical computing systems based on microelectronics with a high degree of integration, industrial automation, medical equipments and robotic instrumentation (e.g., improving sleep for apnea suffers, providing mobile breathing assistance, preventing blood clots in hospitalized patients), etc. In control applications which involve electric drives, it is often necessary to estimate derivatives of measurable states, for example, estimating the speed (ω) and acceleration (ε) from position information (θ_m), which is why in this subchapter four design approaches are proposed and developed: (1) an observer of mth order derivatives with the filtration technique (OFT), (2) an extended Luenberger state observer with θ_m and M_e as inputs (ELSO+Me), (3) an extended Luenberger state observer having only the θ_m as input (ELSO-Me), and (4) a Kalman filter observer (KFO).

The mathematical model (MM) of the three-phase BLDC motor is of the fourth order, having as states variables i_a, i_b, θ_m şi ω_m. In this context, the process dynamics can be divided into two subsystems: *the electromagnetic subsystem (ES)* and *the mechanical subsystem (MS)*. The dynamic equations associated with the two subsystems in state-space form are:

$$ES: \begin{bmatrix} \dot{i}_a \\ \dot{i}_b \end{bmatrix} = \begin{bmatrix} \dfrac{-R}{L-M} & 0 \\ 0 & \dfrac{-R}{L-M} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix} + \begin{bmatrix} \dfrac{2}{3(L-M)} & \dfrac{1}{3(L-M)} \\ \dfrac{-1}{3(L-M)} & \dfrac{1}{3(L-M)} \end{bmatrix} \begin{bmatrix} v_{ab} - e_{ab} \\ v_{bc} - e_{bc} \end{bmatrix}$$

$$Me = \frac{e_a i_a + e_b i_b + e_c i_c}{\omega_m}. \tag{B1.I.1}$$

$$MS: \begin{bmatrix} \dot{\theta}_m \\ \dot{\omega}_m \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & \dfrac{-B}{J_{BLDC}} \end{bmatrix} \begin{bmatrix} \theta_m \\ \omega_m \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{J_{BLDC}} \end{bmatrix} M_e + \begin{bmatrix} 0 \\ \dfrac{-1}{J_{BLDC}} \end{bmatrix} M_{load}.$$

where $x_{ES}=[i_a \ i_b]^T$ (here, T represents matrix transposition) are the states of the ES and $x_{MS}=[\theta_m \ \omega_m]^T$ are the states of the MS. The four design approaches presented below will only use the dynamic model corresponding to the MS.

B1.1. Observer of mth Order Derivatives with Filtration Technique (OTF)

The ideal m^{th} order derivative is characterized by a pure anticipatory dynamic, so it is not practically achievable. A possible solution is to use an nth order low-pass filter for the measured θ_m, connected in series with a pure derivation operator, respecting the constraint m≤n. The values $m = 2$ and $n = 3$ are set due to the fact that $d^2\theta_m/dt^2 = d\omega_m/dt = \varepsilon$, where ε represents the acceleration. Using the substitution $x=[x_1 \ x_2 \ x_3]^T=[\theta_m \ \omega_m \ \varepsilon]^T$ the observable canonical form used in the observer

structure in Fig. B1.1 is:

$$
\begin{bmatrix} \dot{\hat{\theta}}_m \\ \dot{\hat{\omega}}_m \\ \dot{\hat{\varepsilon}} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_0 & -a_1 & -a_2 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \hat{\theta}_m \\ \hat{\omega}_m \\ \hat{\varepsilon} \end{bmatrix}}_{\hat{x}} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ a_0 \end{bmatrix}}_{B} \theta_m \quad \text{or} \quad \bar{\varepsilon} = \frac{a_0 s^2}{s^3 + a_2 s^2 + a_1 s + a_0} \theta_m
$$
(B1.1.1)

Naming this structure an observer is justified due to the fact that Fig. B1.1 shows a correction based on the output estimation error, although for the estimation using the filtering technique this idea was not the starting point. It is expected that the dynamic response of the estimate is slower because through the coefficient a_0 there is only one direct correction term based on the position error. The terms a_1 and a_2 appear as state feedbacks and not the estimation error. The observer design uses the pole placement method. For the imposed poles (real negative ones) $P_\sigma = \{p_1, p_2, p_3\}$ the observer coefficients are:

$$
a_2 = -(p_1 + p_2 + p_3), a_1 = p_1 p_2 + p_1 p_3 + p_2 p_3, a_0 = -p_1 p_2 p_3.
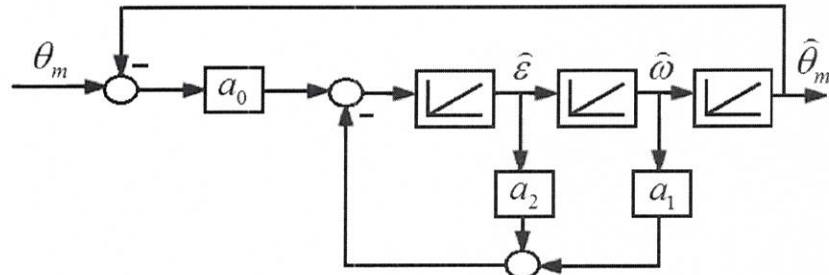$$
(B1.1.2)

Fig. B1.1. OTF block diagram.

B1.2. *Extended Luenberger State Observer with θ_m and M_e as Inputs (ELSO+Me)*

A solution to compute the state estimates $\hat{\omega}, \bar{\varepsilon}$ and $\hat{M}_{Load}$ is to use an extended Luenberger state observer (ELSO) having as predictive correction term the mechanical position error, $\Delta\theta = \theta_m - \hat{\theta}_m$ and as direct input the electromagnetic torque, M_e. For M_{Load} an exogenous model of step signal class is used. In this particular case of ELSO, the state-space equations are (Luenberger, 1966; Andreescu, 1999):

$$
\begin{bmatrix} \dot{\hat{\theta}}_m \\ \dot{\hat{\omega}}_m \\ \dot{\hat{M}}_{Load} \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & \dfrac{-B}{J_{BLDC}} & \dfrac{-1}{J_{BLDC}} \\ 0 & 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \hat{\theta}_m \\ \hat{\omega}_m \\ \hat{M}_{Load} \end{bmatrix}}_{\hat{x}} + \underbrace{\begin{bmatrix} 0 \\ \dfrac{1}{J_{BLDC}} \\ 0 \end{bmatrix}}_{B} \hat{M}_e + \underbrace{\begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix}}_{L} (\theta_m - \hat{\theta}_m),
$$
(B1.2.1)

$$
y = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} \hat{\theta}_m \\ \hat{\omega}_m \\ \hat{M}_{Load} \end{bmatrix}}_{\hat{x}}.
$$

where $x = [\theta_m \ \omega_m \ M_{Load}]^T$ and the parameters l_1, l_2 and l_3 of the observer gain are calculated using the pole placement method, knowing that the pair (A,C) is observable. The ELSO+Me structure is presented in Fig. B1.2 and the observer parameters are:

$$
l_1 = -p_1 + p_2 + p_3 - B/J_{BLDC}.
$$
$$
l_2 = p_1 p_2 + p_1 p_3 + p_2 p_3 - l_1 (B/J_{BLDC}), \quad l_3 = J_{BLDC} p_1 p_2 p_3.
$$
(B1.2.2)

The observer coefficients depend as expected on the MS parameters (especially on J_{BLDC}).

These coefficients determine the dynamics of the observer convergence speed. The idea of using all physical states as observer inputs leads to an observer with improved performance with minimal dynamic delays.
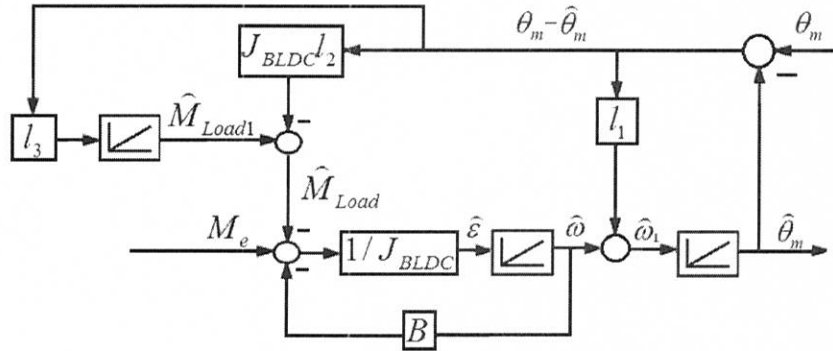

Fig. B1.2. ELSO+M_e block diagram.

B1.3. Extended Luenberger State Observer with θ_m as Input (ELSO-Me)

Another solution to compute the estimates $\hat{\omega}$ and $\hat{\varepsilon}$ is to use an ELSO considering only the mechanical position error term, without any other input. The motion model is chosen with $\varepsilon = const$, a case commonly encountered in constant acceleration with constant electromagnetic torque and in constant steady-state motion regimes. For this case of ELSO-Me, (here, $x = [\theta_m \; \omega_m \; \varepsilon]^T$) the state-space equations are (Luenberger, 1966; Andreescu, 1999):

$$
\underbrace{\begin{bmatrix} \dot{\hat{\theta}}_m \\ \dot{\hat{\omega}}_m \\ \dot{\hat{\varepsilon}} \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} \hat{\theta}_m \\ \hat{\omega}_m \\ \hat{\varepsilon} \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix}}_{l} (\theta_m - \hat{\theta}_m),
$$

$$
y = \underbrace{\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}}_{c} \underbrace{\begin{bmatrix} \hat{\theta}_m \\ \hat{\omega}_m \\ \hat{\varepsilon} \end{bmatrix}}_{x}.
$$

(B1.3.1)

The ELSO-Me block diagram is illustrated in Fig. B1.3. The observer compensator design $L = [l_1 \; l_2 \; l_3]^T$ is also done with the pole placement method and the observer parameters are:

$$
l_1 = -p_1 + p_2 + p_3, l_2 = p_1 p_2 + p_1 p_3 + p_2 p_3, l_3 = -p_1 p_2 p_3.
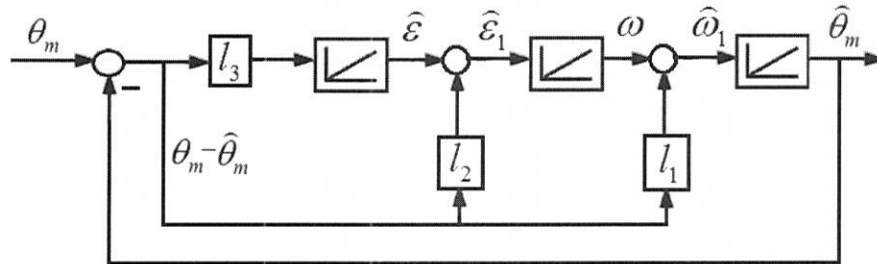$$

(B1.3.2)

Fig. B1.3. ELSO-M_e block diagram.

B1.4. Kalman Filter Observer (KFO)

KFO refers to a probabilistic estimation approach and is the most well-known and commonly used observer for linear systems. Although the KFO in the case of highly nonlinear processes becomes unstable and has severe limitations, when employed for linear processes

provides an efficient approach for estimating the states and thus it also minimizes the mean square error (MSE). Even if an accurate model of the considered system is unknown the filter supports the estimation of past, present and future states (Brown și Hwang, 1996; Lendek et al., 2008; Habibullah și Lu, 2015). As well as the other observers discussed above, KFO will also use the dynamic model corresponding to the MS (here, with C=[1 0] and D=0). The dynamical state-space model can be written as:

$$\dot{x} = A x + B u + G w,$$
$$y = C x + D u + H w + v,$$

(B1.4.1)

where the parameters for matrices A and B are obtained using (B1.1.1), u – the (deterministic) input, w – the white process noise and v – the white measurement noise. Knowing that $E(w) = E(v) = 0$, $E(ww^T) = Q$ (Q – process noise covariance), $E(vv^T) = R$ (R – measurement noise covariance) and $E(wv^T) = N$, we can construct a state estimate $\bar{x}$ that minimizes the steady-state error covariance according to:

$$P = \lim_{t \to \infty} E\left(\{x - \bar{x}\}\{x - \bar{x}\}^T\right)$$

(B1.4.2)

In this context, the equations of optimal Kalman filter-based solution are:

$$\dot{\hat{x}} = A \hat{x} + B u + L(y - C \hat{x} - D u),$$

$$\begin{bmatrix} \hat{y} \\ \hat{x} \end{bmatrix} = \begin{bmatrix} C \\ I \end{bmatrix} \hat{x} + \begin{bmatrix} D \\ 0 \end{bmatrix} u.$$

(B1.4.3)

By solving an algebraic Riccati equation we can determine the filter gain L according to:

$$L = (PC^T + \overline{N})\overline{R}^{-1},$$

(B1.4.4)

where $\overline{R} = R + HN + N^T H^T + HQH^T$ and $\overline{N} = G(QH^T + N)$. The KFO block diagram is presented in Fig. B1.4.
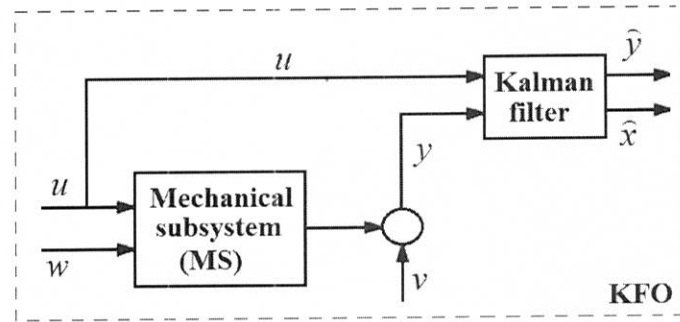

Fig. B1.4. KFO block diagram.

II. The laboratory equipment – Model 220 Industrial Plant Emulator

The development of suitable algorithms to perform the estimation has been the focus of many researchers' attention and for this purpose, in order to estimate state variables from the available measurements, several techniques have been developed and introduced (Brown and Hwang, 1996; Aghannan and Rouchon, 2003; Marx et al., 2007; Lendek et al., 2008; Spurgeon, 2008; Magnis and Petit, 2016). In this context, the paper proposes a controller that can be considered as composed of three subsystems: a subsystem that provides the desired output and from the reference value a feed-forward signal, an observer and a feedback derived from the estimated states. The interesting structure of the controller allows it to be applied for a wide range of design methods. The controller structure and the forms of the equations are exactly the same for systems with one input and one output as well as for systems with multiple inputs and outputs. The same controller structure can be obtained by employing many other design techniques. The defining feature of a state feedback controller and an observer have is the complexity of the controlled

system that determines controller's complexity. As such a system model is actually contained by the controller. Thereby the internal model principle that prescribes that an internal model of the controlled system should be contained in the controller is in this paper exemplified. The laboratory application considered in this subchapter is a complex, nonlinear equipment, ideal for studying automatic control solutions dedicated to the control of modern industrial equipment, such as: transmission shafts, conveyors, rotary tables, machine tools and machines of automated assembly (ECP, 2010; Szedlak-Stinean et al., 2016; Szedlak-Stinean et al., 2017).

Rigid drive dynamics

The first principle equations that describe the system in case of rigid drive dynamics are (ECP, 2010; Szedlak-Stinean et al., 2016):

$$
\begin{cases}
\dot{x}_1 = x_2, \\
\dot{x}_2 = \dfrac{-(c_1 + c_2 g^{-2})x_2}{J_d + J_p g'^{-2} + J_l g^{-2}} + \dfrac{T_D}{J_d + J_p g'^{-2} + J_d g^{-2}}, \\
y = x_1.
\end{cases}
\tag{B1.II.1}
$$

The state-space MM of Model 220 Industrial Plant Emulator with rigid drive dynamics is:

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} =
\begin{bmatrix} 0 & 1 \\ 0 & \dfrac{-(c_1 + c_2 g^{-2})}{J_d + J_p g'^{-2} + J_l g^2} \end{bmatrix}
\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} +
\begin{bmatrix} 0 \\ \dfrac{1}{J_d + J_p g'^{-2} + J_l g^2} \end{bmatrix} T_D,
$$

$$
y = [1 \quad 0][x_1 \quad x_2]^T.
\tag{B1.II.2}
$$

where T_D is the drive torque ($T_D = u$), $x = [x_1 \ x_2]^T = [\theta_1 \ d\theta_1/dt]^T$ is the state vector and y is the output. Considering zero initial conditions, the application of the Laplace transform leads to the following transfer function (t.f.):

$$
\frac{\theta_1(s)}{T_D(s)} = \frac{1/(J_d + J_p g'^{-2} + J_l g^2)}{s[s + (c_1 + c_2 g^{-2})/(J_d + J_p g'^{-2} + J_l g^2)]},
\tag{B1.II.3}
$$

Flexible drive dynamics

The first principle equations that describe the system in case of flexible drive dynamics are (ECP, 2010; Szedlak-Stinean et al., 2016):

$$
\begin{cases}
\dot{x}_1 = x_2, \\
\dot{x}_2 = \dfrac{-(kg^{-2})x_1}{J_{dr}} + \dfrac{-(c_1 + c_{12} g^{-2})x_2}{J_{dr}} + \dfrac{kg^{-1}}{J_{dr}} + \dfrac{c_{12} g^{-1}}{J_{dr}}, \\
\dot{x}_3 = x_4, \\
\dot{x}_4 = \dfrac{(kg^{-1})x_1}{J_l} + \dfrac{(c_{12} g^{-1})x_2}{J_l} + \dfrac{(-k)x_3}{J_l} + \dfrac{-(c_2 + c_{12})x_4}{J_l}, \\
y = x_1.
\end{cases}
\tag{B1.II.4}
$$

The state-space MM of Model 220 Industrial Plant Emulator with flexible drive dynamics is:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \dfrac{-(kg^{-2})}{J_{dr}} & \dfrac{-(c_1 + c_{12}g^{-2})}{J_{dr}} & \dfrac{kg^{-1}}{J_{dr}} & \dfrac{c_{12}g^{-1}}{J_{dr}} \\ 0 & 0 & 0 & 1 \\ \dfrac{kg^{-1}}{J_l} & \dfrac{c_{12}g^{-1}}{J_l} & \dfrac{-k}{J_l} & \dfrac{-(c_2 + c_{12})}{J_l} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{J_d + J_p g^{-2} + J_l g^{2}} \\ 0 \\ 0 \end{bmatrix} T_D, \qquad \text{(B1.II.5)}$$

$$y = [1 \ \ 0 \ \ 0 \ \ 0][x_1 \ \ x_2 \ \ x_3 \ \ x_4]^T.$$

where T_D is the drive torque ($T_D = u$, u is the input), $x = [x_1 \ x_2 \ x_3 \ x_4]^T = [\theta_1 \ d\theta_1/dt \ \theta_2 \ d\theta_2/dt]^T$ is the state vector and y is the output. The following t.f. is attached to (B1.II.5):

$$\frac{\theta_1(s)}{T_D(s)} = \frac{J_l s^2 + (c_2 + c_{12})s + k}{d_4 s^4 + d_3 s^3 + d_2 s^2 + d_1 s}, \qquad \text{(B1.II.6)}$$

where $d_4 = J_{dr}J_l$, $d_3 = J_{dr}(c_2+c_{12}) + J_l(c_1 + c_{12}/g^2)$, $d_2 = J_{dr}k + J_l k/g^2 + c_1 c_2 + c_1 c_{12} + c_{12}c_2/g^2$, $d_1 = c_1 k + c_2 k/g^2$.

B1.5. State feedback and observer-based controller design

In cases where the process states are not accessible for measurements or are only partially accessible for measurements and if the process is observable, then it is possible to estimate its states. For this purpose, state estimators or state observers are utilized. The observability test of the linearized state-space MMs can be done using the matrix:

$$Q_o = [C^T \ \ C^T A \ \ C^T A^2 \ \ C^T A^3 \ \ C^T A^4 ...]^T. \qquad \text{(B1.5.1)}$$

The starting point in order to specify the relations that describe the functioning of a state observer, is the state-space MM corresponding to the process, assumed known, with the form:

$$\dot{x} = A x + B u, \qquad \qquad \text{(B1.5.2)}$$
$$y = C x.$$

The variable that is the target of the control process is the output. Firstly, all components of the state vector are assumed as measured. The feedback is constrained to be linear, so it can be considered as (Åström and Murray, 2009):

$$u = -K x + K_{ref} r \qquad \qquad \text{(B1.5.3)}$$

where r is the reference input, K_{ref} is the feed-forward gain and K is the state feedback gain matrix. To calculate the state feedback gain matrix K, the pole placement method is applied. The closed-loop system obtained when the feedback (B1.5.3) is applied to the system (B1.5.2) is given by:

$$\dot{x} = (A - B K)x + B K_{ref} r. \qquad \qquad \text{(B1.5.4)}$$

The state-space MM corresponding to the state observer has the same structure as the process (B1.5.2) and is completed with a correction relation based on the output error $\tilde{y} = y - \hat{y}$. Consequently, the MM is rewritten in the form (Åström and Murray, 2009):

$$\dot{\hat{x}} = A \hat{x} + B u + L(y - C \hat{x}) = (A - L C)\hat{x} + B u + L y, \qquad \text{(B1.5.5)}$$
$$\hat{y} = C \hat{x},$$

where L is the observer gain.

In order to analyze the observer, the state estimation error is defined as $\tilde{x} = x - \hat{x}$. Differentiating and replacing the expressions of $\dot{x}$ and $\dot{\hat{x}}$ leads to $\dot{\tilde{x}} = (A - LC)\tilde{x}$. The error $\tilde{x}$ will go to zero if the matrix L is chosen such that the matrix has eigenvalues / poles with negative real parts. The appropriate selection of the eigenvalues / poles determines the convergence rate (Åström and Murray, 2009). Taking this into account, the design of the state observer involves solving a poles placement problem and also calculating the parameters of the observer gain. The starting

point in designing the state observer is the expression of the characteristic polynomial:

$$\Delta_{ob}(s) = \det(s\,I - A + L\,C) = s^n + \alpha_{n-1}s^{n-1} + \ldots + \alpha_1 s + \alpha_0. \tag{B1.5.6}$$

By allocating the poles of the observer, the characteristic polynomial $\Delta_{ob}(s)$ is expressed as:

$$\Delta_{ob}(s) = \prod(s - p_{ov}) = s^n + \beta_{n-1}s^{n-1} \ldots + \beta_0 \tag{B1.5.7}$$

Because both the system (B1.5.2) and the observer (B1.5.5) have the same order n, the order of the closed loop system is $2n$. In order to obtain the state feedback observer, the design of the observer as well as the design of the state feedback can be realized separately. The closed-loop system is defined as:

$$\begin{bmatrix} \dot{x} \\ \dot{\tilde{x}} \end{bmatrix} = \begin{bmatrix} A - B\,K & B\,K \\ 0 & A - L\,C \end{bmatrix} \begin{bmatrix} x \\ \tilde{x} \end{bmatrix} + \begin{bmatrix} B\,K_{ref} \\ 0 \end{bmatrix} r. \tag{B1.5.8}$$

Due to the fact that the matrix on the right side is block diagonal, the characteristic polynomial of the closed-loop system has the form:

$$\Delta_{xx}(s) = \det(s\,I - A + B\,K)\det(s\,I - A + L\,C). \tag{B1.5.9}$$

This property is called the separation principle (Åström and Murray, 2009). A schematic diagram of the controller is illustrated in Fig. B1.5. It can be observed that the controller includes a dynamic model of the plant, thus respecting the internal model principle. It can also be noticed that the observer determines the dynamics of the controller. As such, the controller can be regarded as a dynamical system having y as input and u as output:

$$\dot{\bar{x}} = (A - B\,K - L\,C)\bar{x} + L\,y,$$
$$u = -K\,\bar{x} + K_{ref}\,r. \tag{B1.5.10}$$

The t.f. of the controller has the form:

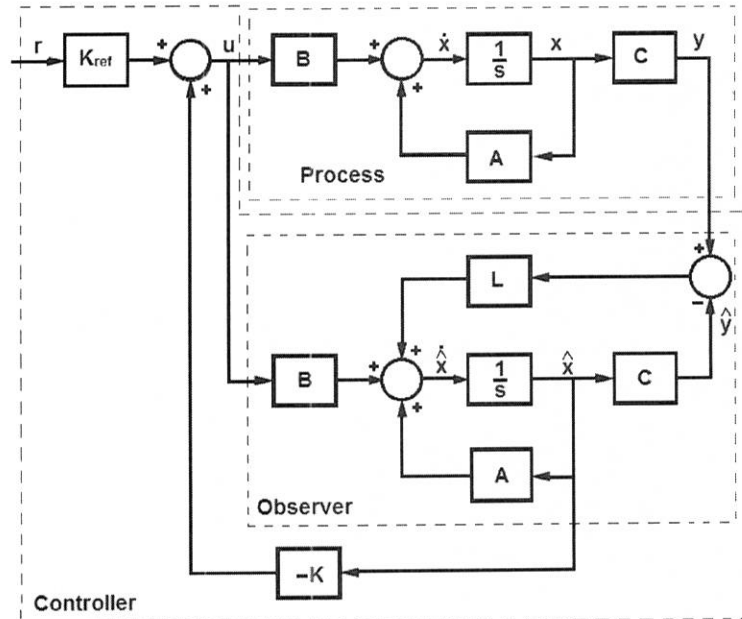$$H_c(s) = K\left[s\,I - A + B\,K + L\,C\right]^{-1}L. \tag{B1.5.11}$$

Fig. B1.5. Schematic diagram of an observer-based controller.

B2. Control structures based on nonlinear observers intended for mechatronic applications.

One of the fundamental problems specific to control systems, of designing observers for nonlinear control systems, was first proposed by Thau (1973). A state constructor (Dorf și Bishop, 2005; Preitl și Precup, 2008; Preitl et al., 2009) is a system having as input the inputs and outputs of the real process, and as output an estimate of the state of this process. In the hypothesis of linearity of the process model, the basic structure of the estimator is always the same, but its realization will depend on the chosen context: continuous or discreet, deterministic or stochastic. If the model is deterministic, the state constructor will be called an observer (or estimator). If the model is a stochastic type, then the constructor will be called a filter. The estimation problem requires the construction, for a deterministic model of the process, of a system defined by its state equation, for which the output provides an estimate of the real state of the process. This estimate implies an error that must tend to zero; when this property is satisfied, the estimator or observer is called asymptotic (Dorf and Bishop, 2005; Preitl and Precup, 2008; Preitl et al., 2009). An observer is very useful for implementing feedback stabilization or feedback regulation of nonlinear control systems due to the fact that it is essentially an estimator for the state of the system. The states estimation problem is of important strategic significance for the process of controlling and monitoring of many technological processes. In order to control and optimize a process, knowing some physical state variables provides useful information. This is the case of many widely diffused process control strategies. Therefore by including an appropriate state estimator, the difficulty caused by the presence of unknown states can be solved. For this reason, the development of suitable algorithms to perform the estimation has been the focus of many researchers attention and for this purpose, in order to estimate state variables from the available measurements, several techniques have been developed and introduced. Over the past three decades there has been significant research done on the nonlinear observer design problem. Depending on the mathematical structure of the process model, there is a wide variety of possible estimators that can be used (Soroush, 1997; Mouyon, 1997; Garcia et al., 2000; Aghannan și Rouchon, 2003; Kinsey and Whitcomb, 2007; Ghanes et al., 2013; Magnis and Petit, 2016).

B2.1. Extended Kalman filter (EKF)

The filtering process aims to determine the estimation of system variables when the environment in which the process takes place presents random perturbations. Two points of view can be used to address this issue: the first one is Wiener's who uses the frequency description and the second one is Kalman's using the temporal description. In both cases, an optimal system (filter) is determined in order to minimize the error variation between the real variable and its estimation (Wiener, 1949; Boulfelfel et al., 1994; Brown and Hwang, 1996). The probabilistic estimation method that is the most well-known and commonly used for linear systems is the Kalman filter (KF) and for the nonlinear systems its extension, the **Extended Kalman Filter (EKF)**. The EKF among other nonlinear ones based on linearization techniques is one of the most widely diffused observers. Because of the fact that the noise model is often unknown and it can only be assumed, wrong noise assumptions can lead to biased or even divergent estimates. The purpose of the observer is to process these measurements and any available informations regarding the initial state of the system and to offer an estimate of the current state of the system. The obtained estimation should improve with additional measurements and, ideally, in the absence of noise, it should converge to the true state value. In estimation theory, EKF is the nonlinear version of the KF that approximately linearizes an estimate of the current mean and covariance. In the case of well-defined transition models, EKF has been considered (Julier and Uhlmann, 2004; Extended Kalman Filter, 2021) a relevant standard in nonlinear state estimation theory, GPS and navigation systems. In terms of EKF, the transition and observation models don't need to be linear functions of the state, but can instead be differentiable functions.

$$x_k = f(x_{k-1}, u_k) + w_k.$$
$$z_k = h(x_k) + v_k. \tag{B2.1.1}$$

where w_k and v_k are the process and observation (measurement) noises, which are both assumed to be zero-mean multivariate Gaussian noises with covariance Q_k and R_k, respectively, and u_k is the control vector. The function f can be used to calculate the predicted state from the previous estimate, and similarly the function h can be used to calculate the predicted measurement from the predicted state. However, f and h cannot be applied directly to the covariance, instead a matrix of partial derivatives is calculated. At each step, the Jacobian is evaluated with the current predicted states. These matrices can be used in the Kalman filter equations. This process essentially linearizes the nonlinear function around the current estimate (Extended Kalman Filter, 2021).

Discrete-time predict and update equations

Predict

- Predict state estimate $\qquad\qquad\qquad \hat{x}_{k|k-1} = f(\hat{x}_{k-1|k-1}, u_k)$ (B2.1.2)

- Predicted covariance estimate $\qquad\qquad P_{k|k-1} = F_k P_{k-1|k-1} F_k^T + Q_k$ (B2.1.3)

Update

- Innovation or measurement residual $\qquad \bar{y}_k = z_k - h(\hat{x}_{k|k-1})$ (B2.1.4)

- Innovation (or residual) covariance $\qquad S_k = H_k P_{k|k-1} H_k^T + R_k$ (B2.1.5)

- Near-optimal Kalman gain $\qquad\qquad\quad K_k = P_{k|k-1} H_k^T S_k^{-1}$ (B2.1.6)

- Updated state estimate $\qquad\qquad\qquad \hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \bar{y}_k$ (B2.1.7)

- Updated covariance estimate $\qquad\qquad P_{k|k} = (I - K_k H_k) P_{k|k-1}$ (B2.1.8)

where the state transition and observation matrices are defined as:

$$F_k = \left.\frac{\partial f}{\partial x}\right|_{\hat{x}_{k-1|k-1}, u_k}, \qquad H_k = \left.\frac{\partial h}{\partial x}\right|_{\hat{x}_{k-1|k-1}} \tag{B2.1.9}$$

<u>*Continuous-time EKF:*</u> unlike the discrete-time EKF, the prediction and update steps are coupled in continuous-time EKF.

<u>Model</u>
$$\dot{x}(t) = f(x(t), u(t)) + w(t), \qquad w(t) \sim N(0, Q(t))$$
$$z(t) = h(x(t)) + v(t), \qquad v(t) \sim N(0, R(t)) \tag{B2.1.10}$$

<u>Initialize</u>
$$\hat{x}(t_0) = E[x(t_0)], P(t_0) = Var[x(t_0)] \tag{B2.1.11}$$

<u>Predict-Update</u>
$$\dot{\hat{x}}(t) = f(\hat{x}(t), u(t)) + K(t)(z(t) - h(\hat{x}(t)))$$
$$\dot{P}(t) = F(t)P(t) + P(t)F(t)^T - K(t)H(t)P(t) + Q(t)$$
$$K(t) = P(t)H(t)^T R(t)^{-1} \tag{B2.1.12}$$
$$F(t) = \left.\frac{\partial f}{\partial x}\right|_{\hat{x}(t), u(t)}, \qquad H(t) = \left.\frac{\partial h}{\partial x}\right|_{\hat{x}(t)}$$

<u>*Discrete-time measurements*</u>: most physical systems are represented as continuous-time models,

while measurements are represented in discrete-time.

Model

$$\dot{x}(t) = f(x(t), u(t)) + w(t), \qquad w(t) \sim N(0, Q(t)) \tag{B2.1.13}$$

$$z_k = h(x_k) + v_k, \qquad v_k \sim N(0, R_k)$$

where $x_k = x(t_k)$.

Initialize

$$\hat{x}_{0|0} = E[x(t_0)], \; P_{0|0} = E\left[(x(t_0) - \hat{x}(t_0))(x(t_0) - \hat{x}(t_0))^T \right] \tag{B2.1.14}$$

Predict

$$\text{solve} \begin{cases} \dot{\bar{x}}(t) = f(\bar{x}(t), u(t)) \\ \dot{P}(t) = F(t)P(t) + P(t)F(t)^T + Q(t) \end{cases} \text{with} \begin{cases} \bar{x}(t_{k-1}) = \hat{x}_{k-1|k-1} \\ \dot{P}(t_{k-1}) = P_{k-1|k-1} \end{cases} \tag{B2.1.15}$$

where $F(t) = \left. \dfrac{\partial f}{\partial x} \right|_{\bar{x}(t), u(t)}$.

Update

$$K_k = P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + R_k)^{-1}$$

$$\hat{x}_{k|k} = \bar{x}_{k|k-1} + K_k (z_k - h(\bar{x}_{k|k-1})) \tag{B2.1.16}$$

$$P_{k|k} = (I - K_k H_k) P_{k|k-1}$$

where $H_k = \left. \dfrac{\partial h}{\partial x} \right|_{\bar{x}_{k|k-1}}$.

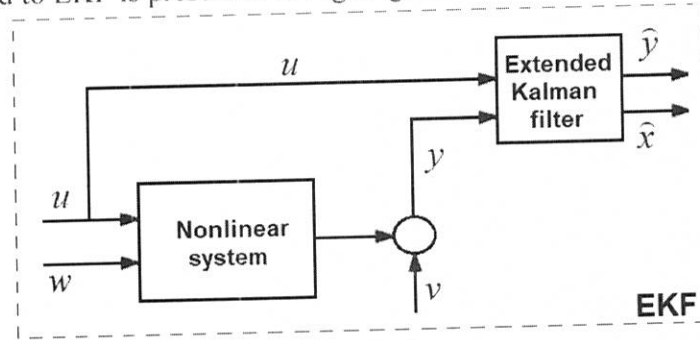The block diagram related to EKF is presented in Fig. Fig. B2.1.

Fig. B2.1. EKF block diagram.

The advantage of using EKF is the lack of stationary error at low speeds. The disadvantage of the method arises from the complexity and large volume of computing, for the implementation the use of processors with high computing power being necessary. Another drawback is the need to fully know the noise sources in the system in order to determine the covariance matrices. For most applications the EFK is still the approach that is more practical and offers the most robustness (Brown and Hwang, 1996; Simon, 2006; Biagiola and Solsona, 2006; Lendek et al., 2008; Beyhan et al., 2013; Habibullah and Lu, 2015).

B2.2. Takagi-Sugeno fuzzy observers (TSFO)

The estimation of states and of the possible uncertain parameters, based on a dynamic system model and a sequence of noisy measurements, is required by a wide range of problems in decision making, control, and monitoring. The design of an observer for a nonlinear system requires the description of the system in a state-space form. There is no generic method yet developed for

the design of an observer valid for all nonlinear system types. From this point of view *fuzzy control* is more pragmatic due to the specific decision-making mode in the command development. To this end, a rather qualitative linguistic characterization of the situation in which the process evolves and of the intervention decision is developed; this decision is then transposed into an adequate quantitative intervention (Preitl şi Precup, 2008; Preitl et al., 2009). Any nonlinear system can be approximated on a compact set with an arbitrary accuracy by the generic nonlinear state-space model that is provided by Takagi-Sugeno (TS) fuzzy systems (Takagi and Sugeno, 1985). The interest of using the approach based on the representation of nonlinear systems through TS models (Takagi and Sugeno, 1985; Taniguchi et al., 2001; Tanaka and Wang, 2001) is build on the fact that once the TS fuzzy models are obtained, some analysis and design tools developed in the theory of linear systems (Huang, 2005; Lee et al., 2013; Li et al., 2014) which facilitates the observation and/or synthesis of the controller for complex nonlinear systems can be used. However, many papers have been done in order to investigate the problem of nonlinear observers' synthesis and its application to the dynamic systems described by fuzzy TS systems. In particular, in (Tanaka et al., 1998, Tanaka and Wang, 2001, Bergsten et al., 2002, Ichalal et al., 2007, Lendek et al., 2010), the problem of the TS fuzzy observer for explicit dynamic models in continuous-time and in discrete-time has been addressed. In implicit cases, there are more continuous-time papers (Taniguchi et al., 2000, Marx et al., 2007, Ilhem et al., 2012; Essabre et al., 2014; Soulami et al., 2015) than for discrete-time (Wang et al., 2012; Estrada-Manzo et al., 2014).

The analysis and design for a general nonlinear system represented by a fuzzy model becomes much easier due to the fact that there are well-established methods and algorithms that can be used to design **fuzzy observers**. For TS fuzzy systems there have been several types of observers developed, such as Thau-Luenberger observers (Beyhan et al., 2013; Tanaka et al., 1998), reduced order observers (Beyhan et al., 2013; Bergsten et al., 2002) and sliding mode observers (Beyhan et al., 2013; Palm şi Bergsten, 2000). The observers design methods generally lead to a linear matrix inequalities (LMIs) feasibility problem. The stability analysis for a large number of rules eventually becomes unsolvable due to the fact that the system complexity grows exponentially with the number of antecedents (Lam et al., 2013). By employing observers, the non-measurable states can be estimated, analyzed, and used for the control of nonlinear systems.

In order to develop a TSFO it is required to have a TS fuzzy model of the nonlinear system subjected to observation. Thus, in this approach, an essential step is the development of a fuzzy model. Generally there are two ways for obtaining fuzzy models, 1. and 2.:

1. Identification (fuzzy modeling) utilizing input-output data, and
2. Derivation from known equations of the nonlinear system.

Using the sector nonlinearity approach ((Kawamoto et al., 1992; Johansen et al., 2000; Mehran, 2008)) is a popular way of obtaining the TS fuzzy models. This approach for the development of fuzzy models was first mentioned in (Kawamoto et al., 1992), and it is based on the following strategy. Considering a simple first order system $\dot{x} = f(x(t))$, with $f(0)=0$, the goal is finding a global sector that fulfills the condition $\dot{x} = f(x(t)) \in \begin{bmatrix} a_1 & a_2 \end{bmatrix} x(t)$. Following this approach ensures the construction of an exact fuzzy model. Due to the fact that it is at times challenging to find a global sector for nonlinear systems, local sector nonlinearity can be taken into consideration. This is a valid approach given the fact that the variables of real-world systems are always bounded.

TS fuzzy models

Model rule i:

IF $z_1(t)$ is M_{i1} and ... and $z_p(t)$ is M_{ip},

$$THEN \begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t), & i = 1,2,...,r \\ y(t) = C_i x(t), & i = 1,2,...,r. \end{cases} \qquad (B2.2.1)$$

where: M_{ij} is the fuzzy sets, r is the number of model rules, $x(t)$ is the state vector, $u(t)$ is the input

vector, $y(t)$ is the output vector, A_i is the square state or system matrices and $z_1(t), ..., z_p(t)$ are premise (scheduling) variables. Each linear equation represented by the relation $A_ix(t) + B_iu(t)$ is labeled as subsystem. Taking into consideration a pair of $(x(t), u(t))$, the final outputs of the TS fuzzy model are given in the following relations (Mehran, 2008):

$$\dot{x}(t) = \frac{\sum_{i=1}^{r} w_i(z(t))\{A_ix(t) + B_iu(t)\}}{\sum_{i=1}^{r} w_i(z(t))} = \sum_{i=1}^{r} h_i(z(t))\{A_ix(t) + B_iu(t)\}$$

(B2.2.2)

$$y(t) = \frac{\sum_{i=1}^{r} w_i(z(t))C_ix(t)}{\sum_{i=1}^{r} w_i(z(t))} = \sum_{i=1}^{r} h_i(z(t))C_ix(t)$$

$$z(t) = \left[z_1(t)z_2(t)...z_p(t) \right],$$

in which for all t we have $\quad w_i(z(t)) = \prod_{j=1}^{p} M_{ij}(z_j(t)),$

(B2.2.3)

$$h_i(z(t)) = \frac{w_i(z(t))}{\sum_{i=1}^{r} w_i(z(t))}$$

Since $\begin{cases} \sum_{i=1}^{r} w_i(z(t)) > 0, \\ w_i(z(t)) \geq 0, \quad\quad i = 1,2,...,r \end{cases}$

(B2.2.4)

we will have for all t $\begin{cases} \sum_{i=1}^{r} h_i(z(t)) = 1, \\ h_i(z(t)) \geq 0, \quad\quad i = 1,2,...,r \end{cases}$

(B2.2.5)

TSFO observer design

The conditions that must be fulfilled for the design of the TSFO are in fact stability conditions to guarantee the stability of the autonomous dynamic systems. The basic stability conditions for an autonomous TS fuzzy system are given in Theorem 1 and Theorem 2 (Beyhan et al., 2013).

Theorem 1: (Tanaka et al., 1998) The equilibrium point $x=0$ of the continuous-time TS fuzzy model described by $\dot{x} = \sum_{i=1}^{r} h_i(z(t))A_i x(t)$ is asymptotically stable, if there exists a positive definite matrix $P = P^T$ so that for $i=1,2,...r$ the following relation is valid:

$$A_i^T P + PA_i < 0$$

(B2.2.6)

The TSFO is described using:

$$\dot{\hat{x}} = \sum_{i=1}^{r} h_i(z(t))\{A_ix(t) + B_iu(t) + L_i(y(t) - \hat{y}(t))\}$$

(B2.2.7)

$$\hat{y}(t) = \sum_{i=1}^{r} h_i(z(t))C_i\hat{x}(t).$$

where L_i, $i=1,2,...r$ are the observer gain parameters. The pairs (A_i, C_i) are assumed to be observable. The goal of the observer is to estimate the states of the model (B2.2.2), which is achieved if the error dynamics $\dot{e} = \dot{x} - \dot{\hat{x}}$ is asymptotically stable. The error dynamics can be written as (Beyhan et al., 2013):

$$\dot{e} = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t))h_j(z(t))(A_i - L_iC_j)\bar{e}$$

(B2.2.8)

The error dynamics of the TSFO in (B2.2.8) can be designed with a desired convergence rate $\alpha > 0$ using Theorem 2.

Theorem 2: (Tanaka et al., 1998) The proper convergence rate of the error dynamics (B2.2.8) is

at least α, if there exists a positive definite matrix $P = P^T$, L_i, $i=1,2,...r$ so that (Beyhan et al., 2013):

$$\mathcal{H}(P(A_i - L_iC_i)) + 2\alpha P < 0,$$
$$\mathcal{H}(G_{ij} + G_{ji}) + 4\alpha P \leq 0,$$

(B2.2.9)

for all pairs *(i, j)*, *i=1, 2,...r, j=1, 2,...r* which there exists $z(t)$ such that încât $h_i(z(t))h_j(z(t))\neq0$. $\mathcal{H}$ denotes the Hermitian matrix, and is defined as $\mathcal{H}(\mathbf{A}) = \mathbf{A} + \mathbf{A}^T$.

B2.3. Sliding mode observers (SMO)

Variable Structure Systems (VSS) are one of the most promising techniques for controlling electrical drives, due to good robustness and performance in case of parametric variations, of perturbations and load variations, respectively due to the compensation of the nonlinearities of the controlled process and their simple implementation. The primary feature that distinguishes the sliding mode systems as an independent class of automatic control systems is the fact that during transient processes the system structure changes. In VSS theory, the most attention is paid to systems with sliding modes. They are based on a specific type of control law that causes a *sliding mode control* or *sliding mode regime* in the system. **Sliding mode observers** use the remarkable VSS properties and are successfully integrated into servo-systems. These observers use the equivalent command to estimate the equivalent disturbance. In general, their structures are closely related to the mathematical models used in the controlled process (Spurgeon, 2008; Mercorelli, 2015; Apaza-Perez et al., 2016). These have the ability to minimize the error between the measured process output and the output of the observer that ensures that the observer produces a set of state estimates that are precisely proportional with the actual output of the process. The sliding mode observer design method consists in determining a switching gain. One restriction is the fact that in order to achieve the estimation, the outputs have to lie on specified sliding surface. In addition, performance is rarely guaranteed, particularly when the outputs are corrupted with noise. Similarly to a KF, the sliding mode observers have attractive noise resilience properties. In comparison to linear observers, the principle advantage that the sliding-mode observers have is that they are insensitive to the unknown inputs while in sliding, and additionally they can be utilized in order to reconstruct unknown inputs which could be a combination of system disturbances, faults or nonlinearities (Spurgeon, 2008; Mercorelli, 2015; Apaza-Perez et al., 2016).

The SMO approach consists of determining a switching variable (Spurgeon, 2008; Shtessel et al., 2014). One constraint is the fact that in order to achieve the estimation, the outputs have to lie on a specified sliding surface. For each individual process, the dimensions of the matrix are $A \in R^{nxn}, B \in R^{nxm}, C \in R^{pxn}$, $p \geq m$, with the matrices B and C of full rank, and the pair (A, C) is observable. In order for the output to appear as component of a new state vector, it is convenient to introduce a coordinate transformation, so, taking this into account we consider the change of coordinates $x \rightarrow T_c x$, $T_c = \begin{bmatrix} N_c^T & C \end{bmatrix}^T$, where the submatrix $N_c \in R^{nx(n-p)}$ spans the null space of C and $\det(T_c) \neq 0$. By applying the change of coordinates $x \rightarrow T_c x$, the matrices of the similar system are:

$$T_cAT_c^{-1} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad T_cB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \quad CT_c^{-1} = \begin{bmatrix} 0 & I_p \end{bmatrix}$$

(B2.3.1)

The canonical form for the nominal system can be rewritten as:

$$\dot{x}_1 = A_{11}x_1 + A_{12}y + B_1u,$$
$$y = A_{21}x_1 + A_{22}y + B_2u,$$

(B2.3.2)

where $T_c x = \begin{bmatrix} x_1 & y \end{bmatrix}^T$. In this context, the SMO proposed by Utkin is (Spurgeon, 2008; Shtessel et al., 2014):

$$\dot{\hat{x}}_1 = A_{11}\hat{x}_1 + A_{12}\hat{y} + B_1 u + L\,v_i,$$
$$\dot{\hat{y}} = A_{21}\hat{x}_1 + A_{22}\hat{y} + B_2 u - v_i,$$

(B2.3.3)

where $(\hat{x}_1, \hat{y})$ are the state and output estimates, $L \in R^{(n-p) \times p}$ is a gain matrix and $v_i = M \operatorname{sgn}(\hat{y}_i - y_i)$ is a discontinuous injection term with $M \in R_+$. The error dynamical system is:

$$\dot{e}_1 = A_{11}e_1 + A_{12}e_y + L\,v_i, \quad e_1 = \hat{x}_1 - x_1,$$
$$\dot{e}_y = A_{21}e_1 + A_{22}e_y - v_i, \quad e_y = \hat{y} - y.$$

(B2.3.4)

An ideal sliding motion will take place on the surface $S = \{(e_1, e_y) : e_y = 0\}$ and after some finite time for all subsequent time $\dot{e}_y = e_y = 0$. The error system defined by (B2.3.4) gives the following relation for the reduced order sliding motion:

$$\dot{e}_1 = (A_{11} + LA_{21})e_1,$$

(B2.3.5)

which depending on L, represents a stable system, so $e_1 \to 0$ and consequently $\hat{x}_1 \to x_1$. The SMO block diagram is given in Fig. B2.3.
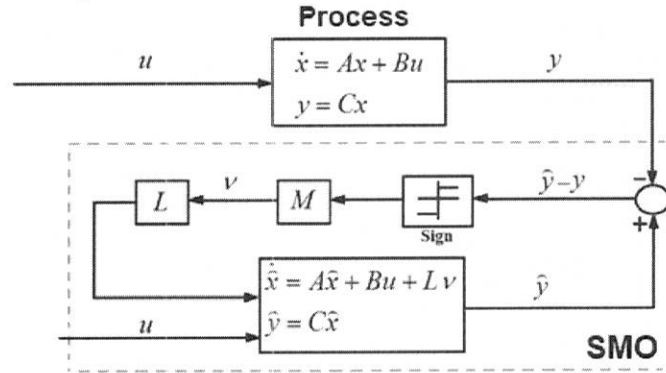

Fig. B2.3. SMO block diagram.

Building upon the previously discussed state-of-the-art, the **main objective of this proposal is to develop the necessary tools, modern control solutions and theoretical framework for later multi-purpose applications related to the observer-based control of mechatronics systems.** The control systems should benefit from the advantages of the continued analytical development and implementation of dynamic model-based nonlinear observers that promise to improve the performance of different types of mechatronics systems. The construction of nonlinear observers still provides an open research field because advances in this area often face many obstacles, such as, for example, the very restrictive conditions that have to be satisfied, uncertainty in the performance and robustness and/or poor estimation results.

Modern control approaches as the **EKF, TSFO** and **SMO** are utilized to estimate states and unknown variables in a nonlinear process in the eventual presence of disturbances or noise. Prior to developing the observer's equation, most researchers developed observers based on the mathematical model of the systems and used the first principles model leading to the fact that model-based approaches are employed by most observer designs. Although the observer gain and its estimation error dynamics are also significant, the evaluation of an observer designs success is based on its ability to estimate the difficult-to-measure states with acceptable convergence rates and with approximately zero estimation errors. Determining the performance of the proposed observers is done by comparing the actual values with those of the estimates. The validation of the proposed

control methods will be carried out in the first phase by means of numerical simulations, and later by means of available laboratory equipment. Further development and improvement of the tuning methods can be undertaken independently. The proposed nonlinear observers will be designed in such a way that they can cope with the nonlinearities and the various restrictions imposed on the various quantities (variables) specific to automatic control systems.

The potential impact to the scientific field may be significant because through new concepts and employed approaches, a new way for the use of highly advanced control designs in mechatronics applications is open, which is very useful for implementing feedback stabilization or feedback regulation of nonlinear control systems. It is clear that nonlinear observer design is still an open area for research, efforts being made to broaden and adapt the proposed techniques in order to widen the classes of nonlinear systems to which they may apply.

The potential impact of the project in the scientific, social, economic or cultural environment is straightforward since the investigated topics can lead to automated tools for controller design and tuning. Although there is a wide range of possibilities for creating new themes for state-of-the-art research, noteworthy is also the impact in the socio-economic environment with directly applicative directions. In the project all mechatronics applications tackled are interdisciplinary and multidisciplinary themselves, with special focus on those applicable cost-effective training systems in the fields of robotics, automation and process control.

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